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While the magnetic field-volume requirements for thruster capable of propelling full-size (2000 ton displacement) submarines appears beyond the present technology, it may be feasible to use superconducting magnets to build high efficiency, internal duct thruster capable of maneuvering small (∿10 ton displacement) submersibles.

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AN EVALUATION OF DIRECT CURRENT ELECTROMAGNETIC PROPULSION IN SEAWATER

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ABSTRACT

Electromagnetic seawater thrusters may be classified in one of three general categories: internal duct dc; external field dc, and peristaltic ac. Internal duct dc thrusters offer the advantages of low magnetic field leakage, simple construction, and potentially high reliability. The most efficient internal duct configuration consists of a converging inlet nozzle and a straight discharge duct. Ideal efficiency calculations based on the one-dimensional Bernoulli equation show that thrusters should be designed with large cross-sectional areas and operate at low discharge velocities. In practice, this may be accomplished by using multiple thruster ducts. Conductivity enhancement, high magnetic fields, and long electrodes also improve efficiency.

While the magnetic field-volume requirements for thruster capable of propelling full-size (2000 ton displacement) submarines appears beyond the present technology, it may be feasible to use superconducting magnets to build high efficiency, internal duct thruster capable of maneuvering small (∿10 ton displacement) submersibles.

TABLE OF CONTENTS

NOMENCLATURE

K_s = $C_D A_s \rho/2$ = constant of proportionality: $D = K_s v_s^2$ $(Nt/(m/s)^2)$

v_s = relative velocity between submerged body and water (m/s)

v_{out} = discharge velocity (m/s)

v_p = velocity in pumping region of duct (m/s)

C_D = drag coefficient (dimensionless)

v_p = velocity in pumping region of duct (m/s)

A_s = effective drag cross-sectional area (m^2)

ρ = density of water (kg/m^3)

σ = electrical conductivity of water (mho/m)

D = drag (newtons)

T = thrust (newtons)

J = duct electrical current density (amp/m^2)

I = duct electrical current (amps)

V = duct voltage (volts)

L = electrode length (meters)

B = duct magnetic field (tesla)

b = one-half electrode spacing (m)

a = one-half duct width along B-field direction (m)

α = aspect ratio = b/a (dimensionless)

A_p = 4ab = cross-sectional area of duct in pumping region (m^2)

A_{out} = cross-sectional area of discharge (m^2)

A_c = cross-sectional area of inlet cone

$\dot{m}$ = mass flow rate thru duct (kg/s)

η = Tv_s/IV = efficiency (dimensionless)

k_1 = derived parameter = v_p/v_s (dimensionless)

P_o = free-stream pressure (Nt/m^2)

P_z = nozzle pressure (Nt/m^2)

P_{in} = pressure at pump inlet (Nt/m^2)

ΔP_z = $P_z - P_o$ = pressure drop through nozzle (Nt/m^2)

ΔP_{in} = $P_o - P_{in}$ = inlet pressure drop (Nt/m^2)

ΔP_p = $P_z - P_{in}$ = pump head (Nt/m^2)

f = factor appearing in voltage, current and efficiency expressions (dimensionless)

N = interaction parameter (dimensionless)

W_f = fluid power (watts)

η_p = propulsive efficiency (dimensionless)

η_e = electrical efficiency (dimensionless)

ε = v_p/v_s (dimensionless)

1.0 INTRODUCTION

This study was undertaken to investigate the practicality of internal duct, direct current electromagnetic propulsion of submersible ocean vessels, ranging from full size submarines down to small maneuverable underwater platforms. The scope of this study is limited to a broad overview of performance characteristics rather than detailed design considerations: the emphasis here is on rapid assessment of trade-offs between major design parameters, such as magnetic field intensity, electrode length and duct size. Performance and efficiency calculations do not, therefore, include minor hydrodynamic and magnetohydrodynamic (MHD) effects, and the resulting curves should be used for general comparisons and trade-off analyses. These curves, for instance, enable the reader to quickly determine overall thrust efficiency as a function of duct area, magnetic field strength, electrode length, and multiple ducting. This information is presented for each of three submersible classes: full, one-third, and one-tenth sizes corresponding to hull diameters of about 10, 3, and 1 meters, respectively.* The assumed drag characteristics for these three sizes are given in Fig. 1.

As will be shown later, overall performance is a strong function of the magnetic field intensity, and the magnitude of field used for the performance curves given here ranges from conventional (0.5 T) to superconducting (5 T) excitation sources. The impact of conductivity enhancement via seeding with hydrochloric acid (HCℓ) is also presented along with estimates of the volume of evolved gases released by electrolysis at the electrode surfaces. Finally, duct dimensions are restricted to reasonable ranges of engineering capability, limited by the volume requirements for the magnetic field. It does not seem reasonable, for instance, to consider a thruster design that requires 10 Tesla (super-

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*See Appendix A-1 for a discussion of hull size, shape, displacement, and drag.

conducting excitation) throughout a duct interior of several hundred cubic meters. Duct dimensions, therefore, range from two to ten meters long and up to one square meter of cross-sectional area.

2.0 CONCLUSIONS AND RECOMMENDATIONS

Electromagnetic thrusters offer a silent, nearly undetectable means of submersible propulsion. Based upon efficiency calculations included in this study though, it seems unlikely that a practical thruster system could be developed for a full size (2000 ton, 10 m hull diameter) submarine. The chief reason for this conclusion lies in the difficulty of establishing high magnetic field intensities throughout the active pumping region contained between the electrodes. A reasonably efficient thruster cannot be designed and built until a superconducting dewar/winding capable of functioning reliably in a submersible environment has been demonstrated. Once this has been achieved, then efficiencies of ten percent or more may be possible for reduced hull sizes of one to three meters diameter, provided that speed is limited to approximately ten knots or less. It may be feasible to use electromagnetic propulsion for manuevering small submersible platforms requiring either modest continuous thrust or occasional bursts of high thrust levels.

In general, efficiency may be improved by:

- minimizing thrust requirements.....small hulls operating at low velocities.
- using large areas, low velocity thrusters.....multiple thrust-ducts are the most practical way of achieving this objective without requiring excessive cross-sectional area per duct.
- operating at high magnetic fields.....preferably at field intensities much greater than that of conventional iron-copper magnetic circuits--hence, superconducting excitation is mandated.
- enhancing conductivity.....seeding seawater with a strong electrolyte such as hydrochloric acid has a pronounced

effect upon overall efficiency. The rate and duration of seeding is limited, of course, by on-board storage capacity.

Development of an electromagnetic thruster should proceed in several stages:

- Define the mission requirements -- range, speed, depth, hull drag, power source, and acceptable levels of magnetic leakage.

- Demonstrate the capability of establishing a high-intensity (~5 Tesla) magnetic field throughout a significant duct volume. This requires design and fabrication of a superconducting dewar/winding capable of functioning reliably in a submersible environment. One possible annular thruster configuration is shown in Figure 33 where a superconducting toroidal* field winding establishes high fields in the pump annulus with relatively low leakage into the surrounding region.

- Construct and test a prototype thruster.

- For military applications, the possibility of detection should receive critical evaluation, and provision should be made to evaluate magnetic field leakage, hydrolysis, and chemical activity (such as pH shift due to seeding) as potential means of detection.

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*Large superconducting toroidal magnets have, in fact, been successfully used in radiotherapy.[9]

3.0 BACKGROUND

3.1 General Description

Electromagnetic (EM) propulsion of sea vessels may be divided into three categories: internal duct dc; external field dc; and peristaltic ac induction. All three types of propulsion have been analyzed and discussed in the literature.(1-4) Here we present a brief description of these three.

The first type of thruster--internal duct dc--consists of electrodes mounted within a propulsion duct such that current (in the conducting fluid) established between the electrodes is perpendicular to a magnetic field established within the same region, but perpendicular to the direction of current flow. (See Fig. 2 for a pictorial diagram.) Interaction of current and magnetic field produces a mutually perpendicular pressure gradient or force directed along the duct axis. If the duct is open at either end, this gradient causes fluid flow and, hence, an axial reaction thrust. While in principle electromagnetic propulsion may be achieved quite easily by passing current between two electrodes located within a magnetic field, in practice the low value of electrical conductivity of seawater places a severe constraint on overall thrust efficiency that may be achieved.

The second type of thruster--external dc field--does not utilize thrust ducts. Instead, exterior electrodes are mounted along the hull and an external magnetic field is established such that interaction of electrode current and field produces external pressure gradients along the submersible's centerline, thereby creating propulsive thrust. Once again, though, the low conductivity of seawater limits ultimate thrust efficiency. A useful system requires superconducting excitation that establishes high magnetic fields throughout large volumes of water:

however, not only are such superconducting magnets expensive and difficult to build, but the resulting unshielded, easily detectible fields are most likely not suitable for military applications.

Finally, the third type of electromagnetic thruster--peristaltic ac induction--utilizes a flexible membrane to separate two fluid regions: one, an annular chamber containing a highly conducting fluid such as liquid metal; the other an inner cylindrical chamber open to the surrounding water. Radial ac magnetic fields induce circumferential ac currents in the outer (highly-conducting) annular chamber and these currents interacting with the induction fields generate pressure waves that impart axial motion to the conducting liquid. This, in turn, distorts the flexible membrane, thereby imparting axial motion to the water core. In short, traveling waves along the membrane squeeze the inner core, squirting water axially, thereby generating reaction thrust. While peristaltic induction may offer attractive efficiency and performance characteristics due to the high conductivity of the enclosed fluid (liquid metal), it does pose significant reliability and safety problems: the efficient liquid metals react chemically with most engineering materials--including water.

In summary, while all three types of electromagnetic propulsion pose significant technical problems, the internal duct design is simpler and probably more reliable than the others, and this work was commissioned specifically for a study of internal duct dc propulsion.

3.2 Performance Calculations

This section deals with specific items relating to performance evaluation, namely:

- choosing a primary measure of performance;
- separating the various interrelated parameters into dependent and independent variables;
- defining ranges of independent parameter variation.

3.2.1 Measure of Performance

Since many interrelated parameters enter into the performance calculations, we must decide upon the key or fundamental relationships that we wish to examine: ultimately, what are we interested in comparing? Useful thrust vs. input power at a fixed cruising speed? Or, cruising range vs. stored energy as a function of hull size? Or, maximum speed vs. electrode size as a function of magnetic field?

As mentioned earlier, internal duct E-M thrusters have an inherently low power conversion efficiency, and this is perhaps a most crucial measure of performance. Most of the results obtained in this study, therefore, are given in terms of ideal* efficiency as speed curves for several hull sizes. Additional curves show the effects of conductivity enhancement (HCℓ seeding) as well as magnetic field intensity upon efficiency. Finally, the last curves show overall input power and thrust power as a function of speed for several hull sizes.

3.2.2 Parameter Specification

Clearly, we are not able to choose independent values for each of the interdependent parameters that influence performance. If, for example, we specify electrode voltage and current, then for a given duct size, the thrust requirement (hull drag) determines magnetic field intensity as a function of speed: all of these quantities cannot be selected independently. Mathematically, we cannot exceed the degrees of freedom that exist between the performance equations.

Our approach, therefore, has been to permit electrode voltage and current to remain unspecified--they are dependent variables whose values are determined as a consequence of other parameter selections. Since dc electrical power sources may be configured to match or nearly

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*Ideal in that electrolysis and hydrodynamic thruster losses are not included (see Sections 4.2, 4.9, and A-1). Magnetic excitation losses, either in terms of ohmic power dissipation (conventional magnet) or refrigeration power (superconducting magnet) are also not included in these calculations.

match most power requirements (e.g., 10 volts at 200 amps or 200 volts at 10 amps), this is a reasonably prudent engineering approach.*

The following parameters are treated as independent variables and must be specified:

- magnetic field
- electrode length
- cross-sectional area of duct
- number of ducts
- fluid conductivity

3.2.3 Range of Parameter Variation

Lastly, the ranges of variation must be specified, and here again we are guided by practical engineering limitations rather than theoretical speculation. All five of the independent variables listed above are related in the sense that choosing one places practical constraints on the others. An extremely high, uniform magnetic field, for instance, cannot be established throughout a large volume. Duct dimensions must reflect this: we cannot seriously discuss duct lengths of hundreds of meters, nor can be consider cross-sectional areas greater than one or two square meters. The following ranges, therefore, were defined and specific calculations made for various combinations of these values:

Variable	Range
• Duct Area	0.25, 0.5, 1 square meters
• Electrode Length	2, 10 meters
• Magnetic Field	0.5, 2, 5 Tesla (2 to 5 T implies superconducting excitation)

---

*It should be noted, however, that designing an actual thruster system may require several iterations wherein the thruster performance equations first determine an approximate power source; then actual power source terminal characteristics - such as would be available from various combinations of, say, submarine batteries - could be used for design modifications. An important part of this iteration would be variations in the aspect ratio, as discussed later in Section 4.9.

- Number of Ducts .... 1, 4, 9, 16 (Each of the multiple ducts has the same cross-sectional area and electrode length. That is, one duct is not subdivided into multiple channels such that the total thrust area remains constant, but rather the total duct area increases by factors of 4, 9, and 16.)

- Conductivity .... 4 mhos, the nominal conductivity of seawater. The effects of seeding with HCL were calculated by doubling the conductivity to 8 mhos. This corresponds to a 0.5% by weight solution of HCℓ.

4.0 THRUSTER PERFORMANCE CALCULATIONS

4.1 Thruster Model

Water-jet thrusters work on the principle of momentum conservation: an increase in stream momentum from intake to discharge produces a net reaction thrust. As shown in the pictorial sketch of Fig. 2, the electromagnetic thruster is composed of three sections: 1) converging inlet nozzle; 2) electrode-magnetic field region; 3) converging discharge nozzle. Definitions of the electrode (or pumping) region dimensions are also given in Fig. 2.

The converging inlet nozzle provides a smooth transition from the free-stream region immediately ahead of the intake to the internal pumping region, and if we assume that at a fixed uniform cruising speed, v_s*, the inlet area is just large enough to provide the volumetric flow required, then no power will be expended drawing water into the inlet from ahead of the inlet plane. Matching the converging inlet opening to cruising speed also has the advantage of minimizing the thruster's displacement drag in that incident flow is directed into the duct, rather than around it.

The electrode or active pumping region consists of parallel electrodes mounted along the channel walls with a magnetic field established normal to the current-flow direction. The rectangular cross-sectional area, A_p, is assumed to be constant throughout the electrode region.

The third and last section of the thruster is the converging nozzle, which directs flow from the pumping region to a discharge orifice.

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*See Nomenclature for symbol definitions.

4.2 Assumptions

In the calculations that follow, the following assumptions have been invoked unless otherwise noted:

- Thrust = velocity-squared drag

  The thrust required to maintain a fixed speed is equal to total drag. Total hull drag, we assume, may be described by one equivalent drag coefficient that includes both frictional and displacement components. Total drag is then proportional to velocity squared times the cross-sectional area (see Fig. 1 for curves of drag vs. speed. Also see Appendix 1 for a comparison with more accurate drag calculations).

- Negligible thruster drag

  Based upon a thruster design that utilizes a converging inlet nozzle, this is a reasonable assumption (see Appendix 1 for details).

- Negligible electrode-electrolysis losses (electrolysis voltage << pump voltage). The validity of this assumption depends to a large extent upon the aspect ratio, see Section 4.9.

- Nozzle inlet velocity = relative velocity between submersible and surrounding water

- Absence of cavitation within the inlet nozzle

  A convergent inlet nozzle forces a static pressure drop with respect to the inlet or free stream pressure. Assuming no cavitation is equivalent to assuming that the submersible depth is great enough such that the absolute pressure at the pump inlet (P_{in}) is greater than the vapor pressure of water (several psia).

4.3 Thruster Performance Equations

4.3.1 Thrust and Velocity Relationships

(See Fig. 2 and the Nomenclature for symbol definitions.) From the conservation of momentum, we have

thrust = outlet momentum - inlet momentum

or

$$T = \dot{m}(v_{out} - v_{in}) \tag{1}$$

where the main flow, $\dot{m}$, is given by

$$\dot{m} = \rho v_p A_p = \rho v_s A_{in} = \rho v_{out} A_{out} \tag{2}$$

Combining these two equations, we can express thrust as

$$T = \dot{m}\, v_p \left(\frac{A_p}{A_{out}} - \frac{v_s}{v_p} \right) = \rho A_p v_p^{\,2} \left(\frac{A_p}{A_{out}} - \frac{v_s}{v_p} \right) \tag{3}$$

But thrust developed by the duct is equal to the submersible's drag force

$$D = (C_D A_s \rho / 2) v_s^{\,2} = K_s v_s^{\,2} \tag{4}$$

or

$$T = K_s v_s^{\,2} \tag{5}$$

where $K_s = C_D A_s \rho / 2$

Combining Eqs. (3) and (5), we get for k_1 the ratio of internal duct velocity to inlet velocity

$$k_1 = \frac{v_p}{v_s} = \frac{A_{out}}{2A_p} [1 + (1 + 4K_s/\rho A_{out})^{1/2}] \qquad (6)$$

Also, note that from Eq. (2), $v_p A_p = v_s A_{in}$, so that $k_1 = A_{in}/A_p$ = ratio of inlet area to area of pumping region.

4.3.2 Pressure Relationships (neglecting hydrodynamic losses)

Inlet

Applying the Bernoulli Equation, we have

$$P_o + 1/2\ \rho\ v_s^2 = P_{in} + 1/2\ \rho\ v_p^2 \qquad (7)$$

Rearranging and using the pressure differential ΔP_{in} yields

$$\Delta P_{in} = P_o - P_{in} = 1/2\ \rho\ (v_p^2 - v_s^2)$$

or

$$\Delta P_{in} = 1/2\ \rho\ v_p^2\ (1 - (v_s/v_p)^2) \qquad (8)$$

Exit Nozzle

Again, applying the Bernoulli Equation we have

$$P_z + 1/2\ \rho\ v_p^2 = P_o + 1/2\ \rho\ v_{out}^2 \qquad (9)$$

then the nozzle pressure drop is

$$\Delta P_z = P_z - P_o = 1/2\ \rho\ (v_{out}^2 - v_p^2)$$

or using Eq. (2) to express v_{out} in terms of v_p, we get

$$\Delta P_z = 1/2\ \rho\ v_p^2 \left[\left(\frac{A_p}{A_{out}} \right)^2 - 1 \right] \tag{10}$$

Pump

$$\text{pump head} = \Delta P_p = P_z - P_{in} = \Delta P_z + \Delta P_{in} \tag{11}$$

Substituting from Eqs. (8) and (10):

$$\Delta P_p = 1/2\ \rho\ v_p^2 \left[\left(\frac{A_p}{A_{out}} \right)^2 - \left(\frac{v_s}{v_p} \right)^2 \right] \tag{12}$$

But from Eq. (6), $v_p = k_1 v_s$ and we can express the pump head in terms of the relative velocity, v_s:

$$\Delta P_p = 1/2\ \rho\ v_s^2 \left[k_1^2 \left(\frac{A_p}{A_{out}} \right)^2 - 1 \right] \tag{13}$$

This is the pump head or pressure differential that must be generated electromagnetically in order to sustain the relative velocity v_s. If the pump pressure satisfies this requirement, then thrust = drag at speed v_s.

4.3.3 Electromagnetic Pressure

Neglecting electric and magnetic fringe effects near the electrode edges, we have:*

$$\Delta P_p = JBL = \sigma BL \left(\frac{V}{2b} - v_p B \right) \tag{14}$$

*See Appendix A-2 for a derivation of this equation.

4.3.4 Voltage and Current

Substituting for ΔP_p from Eq. (13) and solving for the electrode voltage, V, we get

$$V = \frac{\rho v_s^2 bf}{\sigma BL} + 2Bv_s k_1 b \tag{15}$$

where $f = [(k_1 A_p/A_{out})^2 - 1]$.

Now we can use Eqs. (13), (14), and (15) to solve for the current density, J, and hence the total current, I (= 2aIJ):

$$I = 2aL\sigma \left(\frac{V}{2b} - v_p B\right)$$

$$I = \frac{a \rho v_s^2}{B} ((k_1 A_p/A_{out})^2 - 1)$$

or

$$I = a \rho v_s^2 f/B \tag{16}$$

where f is the dimensionless parameter given above.

4.3.5 Efficiency

Pumping efficiency, η, is the ratio of thrust power to electrical input power

$$\eta = T v_s/IV \tag{17}$$

Substituting from Eqs. (5), (15), and (16), we have

$$\eta = 2K_s/[A_p \rho f k_1(f/(2k_1^2 N) + 1)] \tag{18}$$

where again $f = [(k_1 A_p/A_{out})^2 - 1]$ and we introduce the interaction parameter N, a measure of the interaction between electromagnetic and inertial body forces in the fluid, defined as

$$N = \sigma B^2 L/\rho\, v_p \tag{19}$$

Efficiency may also be expressed in terms of the propulsive and electrical efficiencies, which are defined, respectively, as

$$\eta_p = \frac{T v_s}{W_f} \tag{20}$$

$$\eta_e = \frac{W_f}{I \cdot V} \tag{21}$$

where the fluid power, $W_f = v_p \Delta P_p A_p$.

Substituting from the previous expressions for these quantities, we have

$$\eta = \eta_p \cdot \eta_e \tag{22}$$

with

$$\eta_p = \frac{2}{k_1 A_p/A_{out} + 1} \tag{23}$$

$$\eta_e = \left[1 + \frac{1}{2N}\left(\frac{A_p}{A_{out}}\right)^2 - \frac{1}{k_1^2}\right]^{-1} \tag{24}$$

From Eq. (24) we note that efficiency increases as the nozzle discharge area A_{out} approaches the pump area, A_p. To maximize overall efficiency, then, we set $A_{out} = A_p$. This corresponds to the maximum cross-sectional discharge area consistent with no nozzle cavitation, and the governing equations become:

- Velocity ratio:

$$k_1 = \frac{v_p}{v_s} = \frac{A_{in}}{A_p} = (1/2)\left[1 + (1 + 4K_s/nA_p)^{1/2}\right] \tag{25}$$

- Electrode voltage:

$$V = 2Bv_s k_1 b\ (f/(2k_1^2 N) + 1) \qquad (26)$$

where $f = (k_1^2 - 1)$ and

N = interaction parameter defined above.

- Electrode current:

$$I = a\ \rho\ v_s^2\ f/B \qquad (27)$$

- Efficiency:

$$\eta_p = 2/(k_1 + 1)$$

$$\eta_e = [1 + \frac{1}{2N}\ (1 - k_1^{-2})]^{-1} \qquad (28)$$

where overall efficiency $\eta = \eta_p \cdot \eta_e$.

Equations (25) through (28) provide a basis for evaluating performance of a single duct propulsion system.

4.3.6 Multiple Duct Performance

The single duct equations may be adapted to multiple duct analysis simply by altering the thrust coefficient, K_s. Let $K_s' = K_s/n$ = thrust requirement for each of n ducts. Then we get the following multiple duct relationships:

- per duct velocity coefficient:

$$k_1' = v_p'/v_s = (1/2)\ [1 + (1 + 4K_s'/\rho\ A_p')^{1/2}] \qquad (6')$$

where A_p' = area per duct

- electrode voltage:

$$V' = 2Bv_s k_1' b' (f'/(2k_1'^2 \cdot N') + 1) \tag{26'}$$

where $f' = k_1'^2 - 1$, $N' = \sigma B^2 L/\rho v_p'$

- electrode current:

$$I' = a' \rho v_s^2 f'/B \tag{27'}$$

- efficiency:

$$\eta' = [2/(k_1' + 1)] \cdot [1 + (2N')^{-1} (1 + k_1'^{-2})]^{-1} \tag{28'}$$

Suppose, then, that we compare the performance of n parallel ducts to a single duct where the combined parallel duct pumping area is equal to that of the single duct (that is, $A_p' = A_p/\sqrt{n}$, $a' = a/\sqrt{n}$, $b' = b/\sqrt{n}$) then:

$$k_1' = k_1,\ f' = f,\ N' = N$$

$$I' = I/\sqrt{n},\ V' = V\sqrt{n}$$

$$\eta' = \eta$$

And we see that there is no change in efficiency. If, however, we retain the same area per duct, but add n parallel ducts ($A_p' = A_p$, $a' = a$, $b' = b$) then the velocity ratio k_1' is given by:

$$k_1' = (1/2)\ [1 + (1 + 4K_s/\rho n A_p)^{1/2}] \tag{29}$$

In this case, $k_1' < k_1$, due to the "n" factor appearing in the denominator. Also, we find $f' < f$ and $N' > N$. Substituting these inequalities into Eq. (28'), we find that the multiple duct efficiency increases ($\eta' > \eta$). Thus, increasing the total thrust area by adding parallel ducts decreases the inlet-to-pump velocity ratio and increases efficiency.

4.4 Efficiency Plots

Efficiency plots based upon Eq. (28') are given in Figs. 3 to 26, where calculated, ideal efficiencies (based upon the assumptions discussed earlier) are plotted as a function of submersible velocity, v_s. Each plot contains a set of curves obtained using drag characteristics for three hull sizes, as given in Fig. 1. For each hull size, single duct efficiency is plotted along with several curves showing the effect of four, nine, and sixteen multiple ducts. The figures are arranged in three groups of eight, each group corresponding to ascending magnetic field intensities of 0.5, 2, and 5 Tesla, respectively. Within each group, electrode lengths of 2 and 10 meters are combined with duct areas of .25, .5, .75 and 1.0 square meters. As may be seen from the curves, efficiencies vary from less than .01 to over 99 percent. It must be emphasized that these plots are useful for comparing and indicating sensitivity of ideal performance to parameter variations--all of these combinations are not necessarily realistic. Sixteen thrusters, with a combined cross-sectional area of sixteen square meters propelling a one-meter diameter hull does not, for instance, represent a realistic configuration. But one such thruster propelling this size hull may be entirely feasible.

Not surprisingly, these plots show that efficiency improves as electrode length, magnetic field, duct area, and the number of ducts increase. Low thrust requirements due to less drag (smaller hull sizes) also improves efficiency.

4.5 Conductivity Enhancement: $HC\ell$ Seeding

As indicated by the performance equations derived in Section 4.3, electrical losses diminish and thrust efficiency increases as fluid conductivity increases. The conductivity of seawater, which has a nominal value of about 4 mhos/m, increases significantly with the addition of a small percentage of hydrochloric acid (HCL). Neglecting possible increases in electrode corrosion rates, conductivity enhancement in a thruster may be accomplished by injecting HCL into the inlet nozzle where mixing and diffusion tends to create a uniform mixture prior to the electrode region. While from an efficiency consideration there is strong incentive to inject large amounts of HCL, the benefits of conductivity enhancement must be balanced against the limitations of solute storage capacity on board the submersible. As a compromise between these conflicting requirements, we have arbitrarily chosen a 0.5% solution (.14 molar concentration of HCL), which effectively doubles the bulk conductivity of seawater from 4 mhos/m to 8 mhos/m.[6] Figs. 27-29 show the effects of conductivity enhancement upon ideal efficiencies for full, third, and tenth size hulls, respectively. Note that the basic thruster-duct size (2 m x .25 m^2) is identical in all three cases but that the number of ducts attached to each hull varies from: sixteen ducts, full-size hull; nine ducts, third-size hull; four ducts, tenth-size hull. These configurations are more representative of actual design possibilities than, say, a fixed number of ducts for all three sizes.

Seeding flow rates shown on the plots vary from 9400 gpm at 20 knots, full-size hull to 71 gpm at 2 knots, tenth-size hull. In general, efficiency increases somewhat less than the factor of two increase in conductivity.

4.6 Sensitivity of Efficiency to Magnetic Field Strength

Ideal efficiencies for the three hull sizes and duct combinations (described in Section 4.5) are plotted in Fig. 30 for a fixed speed of 5 KTS and magnetic field variations from .5 to 5 Tesla. Although data for these plots are contained in the basic efficiencies curves given in Figs. 2 to 26, Fig. 30 more clearly illustrates the magnetic field dependence where an order of magnitude change in field intensity yields efficiency changes of nearly two orders of magnitude. As shown in the figure, conventional excitation is limited to less than 2 Tesla due to saturation of ferromagnetic materials used in magnetic circuits.

4.7 Power Requirements Vs. Speed

Total thrust power and electrical input power curves over a speed range of 2 to 20 knots are shown in Fig. 31 for the duct-hull combinations described in Section 4.5. These plots, which are based on a 2 T magnetic field intensity, underline the importance of low speed operation: doubling the cruising speed from 5 to 10 KTS, for instance, increases the total input power requirement by a factor of ten.

4.8 Efficiency as a Function of Interaction Parameter

The electrical and propulsive efficiency expressions given by Eqs. 28 and 29 may be greatly simplified by substituting some typical values for the interaction parameter. Since the order of magnitude is determined mainly by the σ/ρ ratio ($\approx 4 \cdot 10^{-4}$) in the expression $N = \sigma B^2 h/\rho v_p$, we can safely assume $N \ll 1$, or N is less than .01. And if the velocity ratio k_1 is much greater than 1 ($k_1 \gg 1$), then from Eqs. 28 and 29 the overall efficiency $\eta < (.5)(2N)$ or $\eta < N$. That is, the efficiency is less than one percent. If, on the other hand, we assume $k_1 = 1+\varepsilon$ where $\varepsilon \ll 1$ (that is, the velocity differential $v_p - v_s \approx \varepsilon v_s$), then

$$\eta_p' = \frac{2}{2+\varepsilon} = \frac{1}{1+\varepsilon/2} \approx 1 - \varepsilon/2 \qquad (30)$$

and

$$\eta_e' = \frac{1}{1+(\varepsilon/N)} \qquad (31)$$

These expressions indicate that reasonable efficiencies may be achieved, provided that ε, the velocity differential, is of the same order as the interaction parameter, N. This is illustrated by the curve in Fig. 32.

In addition to efficiency, though, a practical design must include other considerations such as: electrode voltage and current, and overall pump dimensions. While simply meeting the $\varepsilon \approx N$ criterion does not guarantee a practical design, the performance expressions discussed here do indicate the sensitivity of overall efficiency to the v_p/v_s ratio.

4.9 Performance as a Function of Aspect Ratio

Although overall ideal efficiency is independent of aspect ratio, actual efficiency will depend to some extent on internal shape of the thruster ducts. This arises for two reasons:

- Viscous drag increases with wetted surface area. A square duct (α=1), therefore has minimum viscous drag. (See Appendix 1 for a detailed discussion of duct drag.)
- Electrolysis voltage remains nearly constant over a wide range of operating current densities.* The ratio between this fixed voltage and the total electrode voltage, though, diminishes as the aspect ratio increases. That is, a duct cross section elongated in the direction of electrode separation requires a higher pump voltage and electrolysis losses become a small fraction of total electrical losses as the elongation increases.

Hence, we have two conflicting requirements, one for a unity aspect ratio in order to reduce viscous drag, the other for a high aspect ratio to minimize electrolysis losses. A careful trade-off analysis must be included in actual thruster designs.

A conventional copper-iron magnetic circuit will produce a more uniform field with reduced excitation losses as the aspect ratio increases. This, too, must be factored into the overall design.

---

*The electrolysis potential at both the anodic and cathodic electrodes is ≈ 2 volts. Since this value is weakly dependent upon current density and strongly dependent on electro-chemical effects, it is realistic to assume a total electrolysis voltage of between five and ten volts.

5.0 GAS EVOLUTION THROUGH ELECTROLYSIS

Dissociation of water at the cathode discharges hydrogen gas along the electrode: two faradays of charge generates one mole (or 22.4 l of H_2 at STP) so that:

$$Q_{H_2} = \text{hydrogen discharge rate} = 5.210^{-3}\ I_{kA}\ \text{l/s} \qquad (32)$$

This, of course, mixes with the thrust-exit stream of water and eventually bubbles to the surface. Using values for discharge flow and electrode currents associated, for example, with the full-hull size curves of Fig. 3, we have (at 20 KTS):

$$\text{thrust stream per duct} = 4.4 \times 10^3\ \text{l/s}$$

$$\text{electrode current/duct} = 104\ \text{kA}$$

then

$$Q_{H_2} = .54\ \text{l/s}$$

and we find an extremely low hydrogen-to-water volume ratio. Evolved hydrogen gas along the electrode surfaces should not hinder electrode performance, nor present a noticeable bubble or foam wake at the surface. Also, it may be possible to construct a porous electrode that captures the evolved gases. Final verification of these conclusions and possibilities must be made experimentally.

6.0 PRELIMINARY DESIGN PROCEDURE

Designing an electromagnetic thruster requires many iterations through the performance equations in order to balance efficiency, physical size and magnetic field intensity with desirable electrical characteristics. The following procedure is offered as a plausible design approach:

(1) Using the known drag vs. speed requirements (K_s) for a particular submersible and assuming a fixed number of thruster ducts of area A_p, find the velocity coefficient, k_1 from Eq. 29. This determines the inlet-to-pump cross-sectional ratio (A_{in}/A_p).

(2) Using Eqs. 26', 27', and 28', calculate voltage, cu.rent, and efficiency at various combinations of the following parameters:

- aspect ratio (α)
- pump length (L)
- magnetic field (B)
- velocity (v_s)

(3) Repeat steps (1) and (2) over the range of interest for:

- drag
- cross-sectional area per thrust-duct
- total number of thrusters

This procedure may be refined by including:

- duct drag (see Appendix A-1)
- electrolysis voltage (see Section 4.9)
- electromagnetic fringe effects
- inlet/outlet hydrodynamic losses

(These last two items should remain second order effects and are not pursued here.)

ACKNOWLEDGMENTS

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Electrotechnology Department
Chemical Sciences Division

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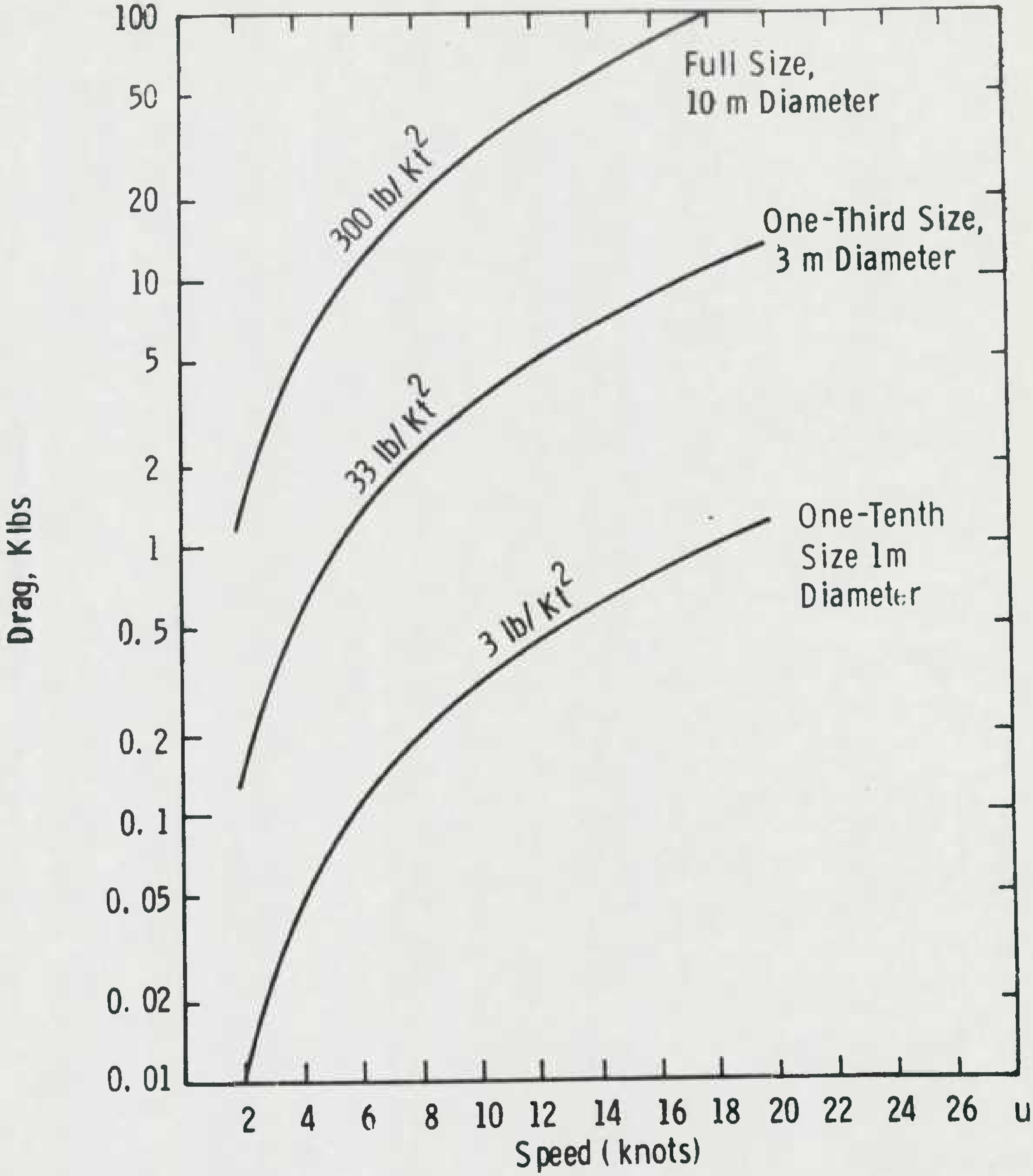

Fig. 1 — Drag characteristics used for performance calculations

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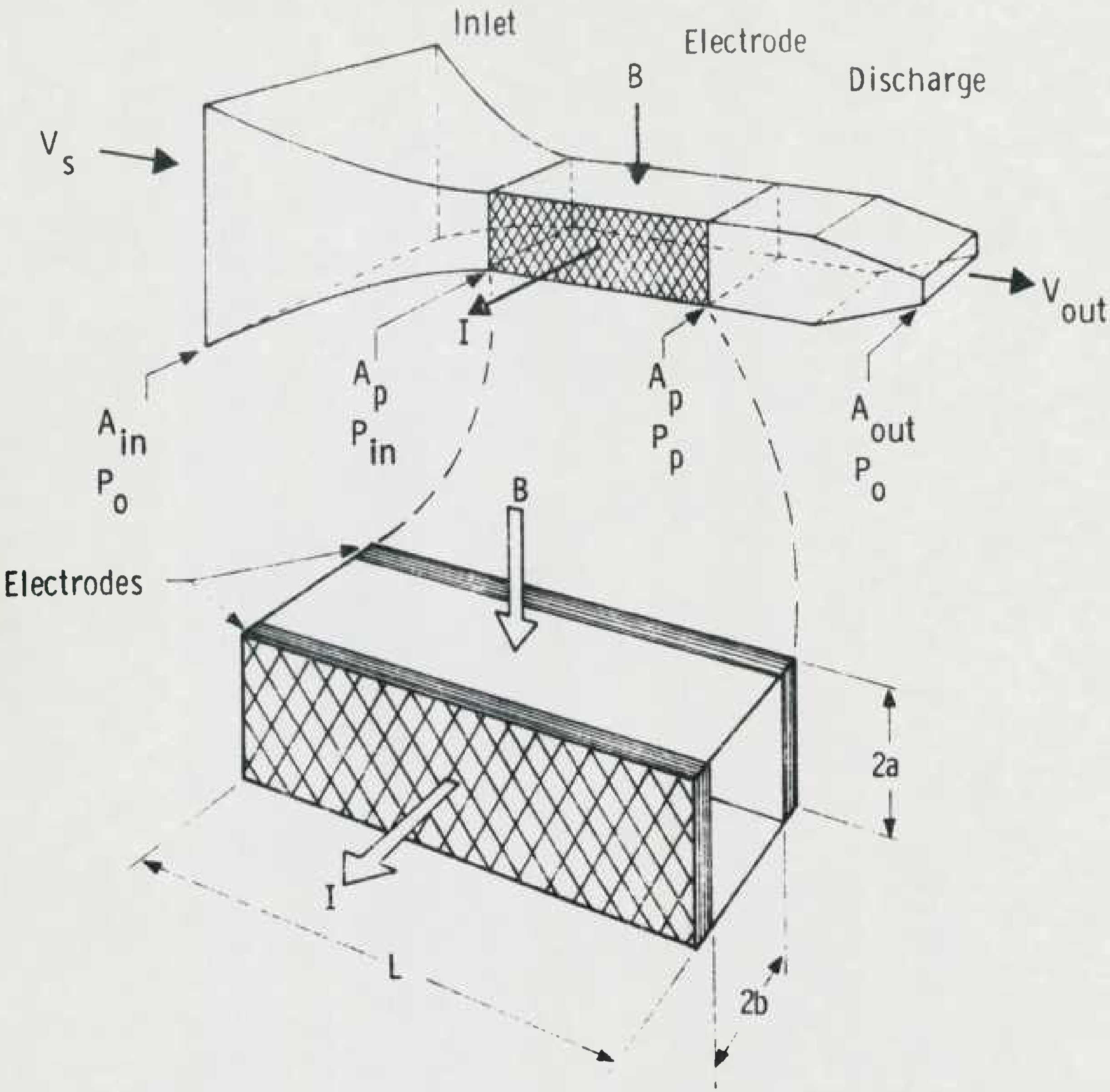
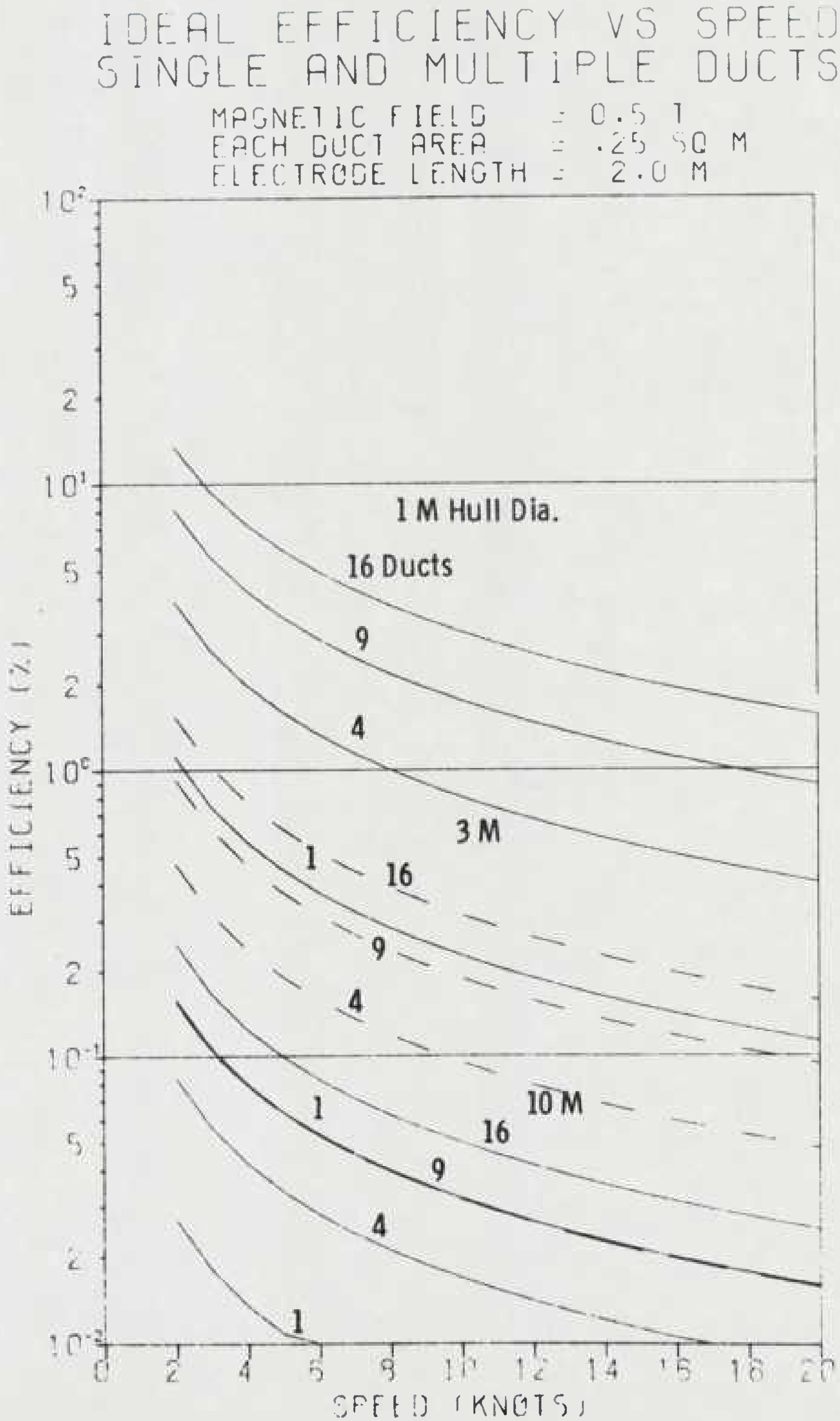


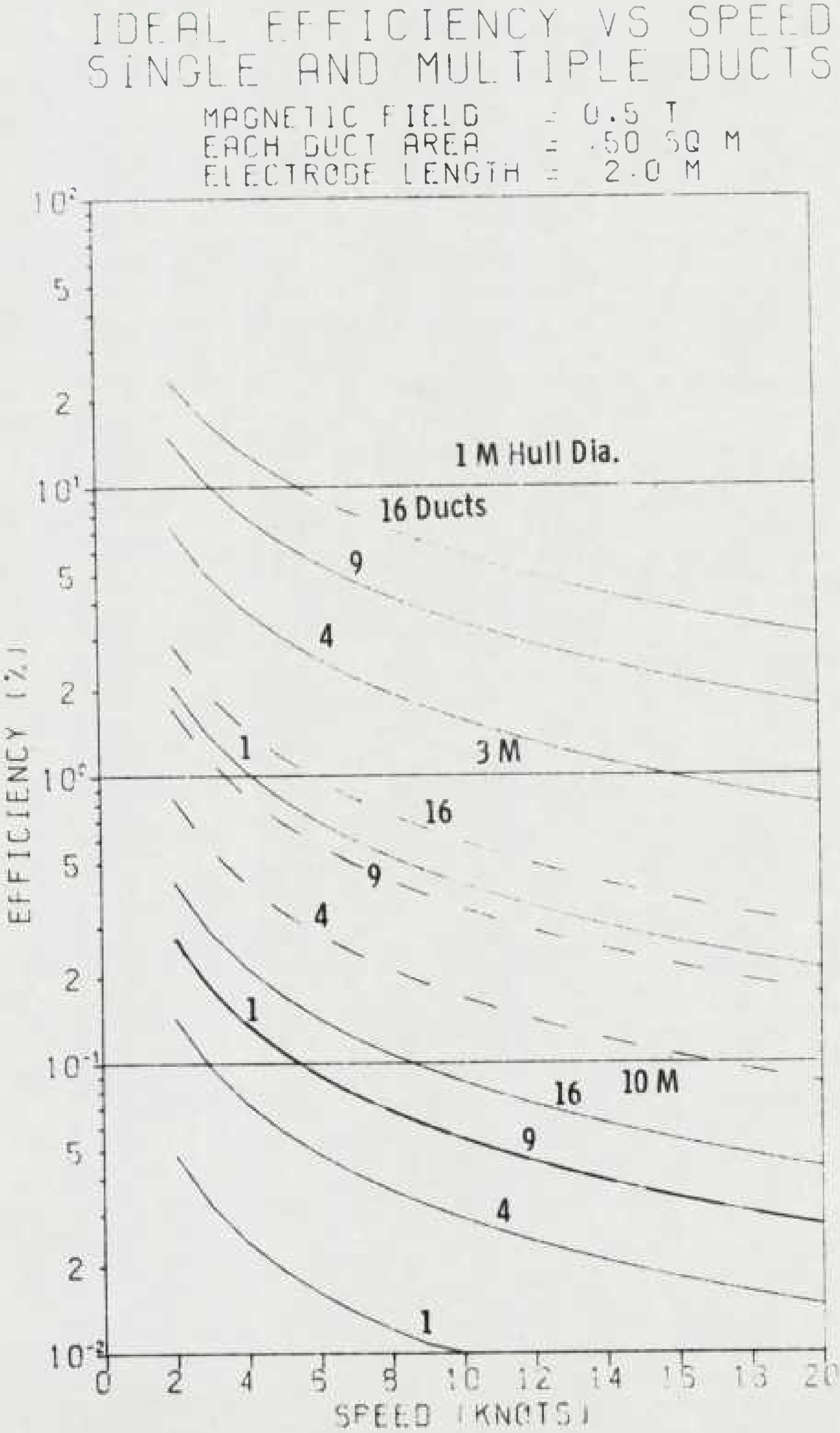
Fig. 2 – Pictorial diagram of internal duct thruster with converging inlet and outlet nozzles. Details of electrode region showing configuration used for duct analysis

FIGURE 3



IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 0.5 T
EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 2.0 M
EFFICIENCY (%)
SPEED (KNOTS)
1 M Hull Dia.
16 Ducts
9
4
3 M
1
16
9
4
10 M
1
16
9
4
1

FIGURE 4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SINGLE AND MULTIPLE DUCTS
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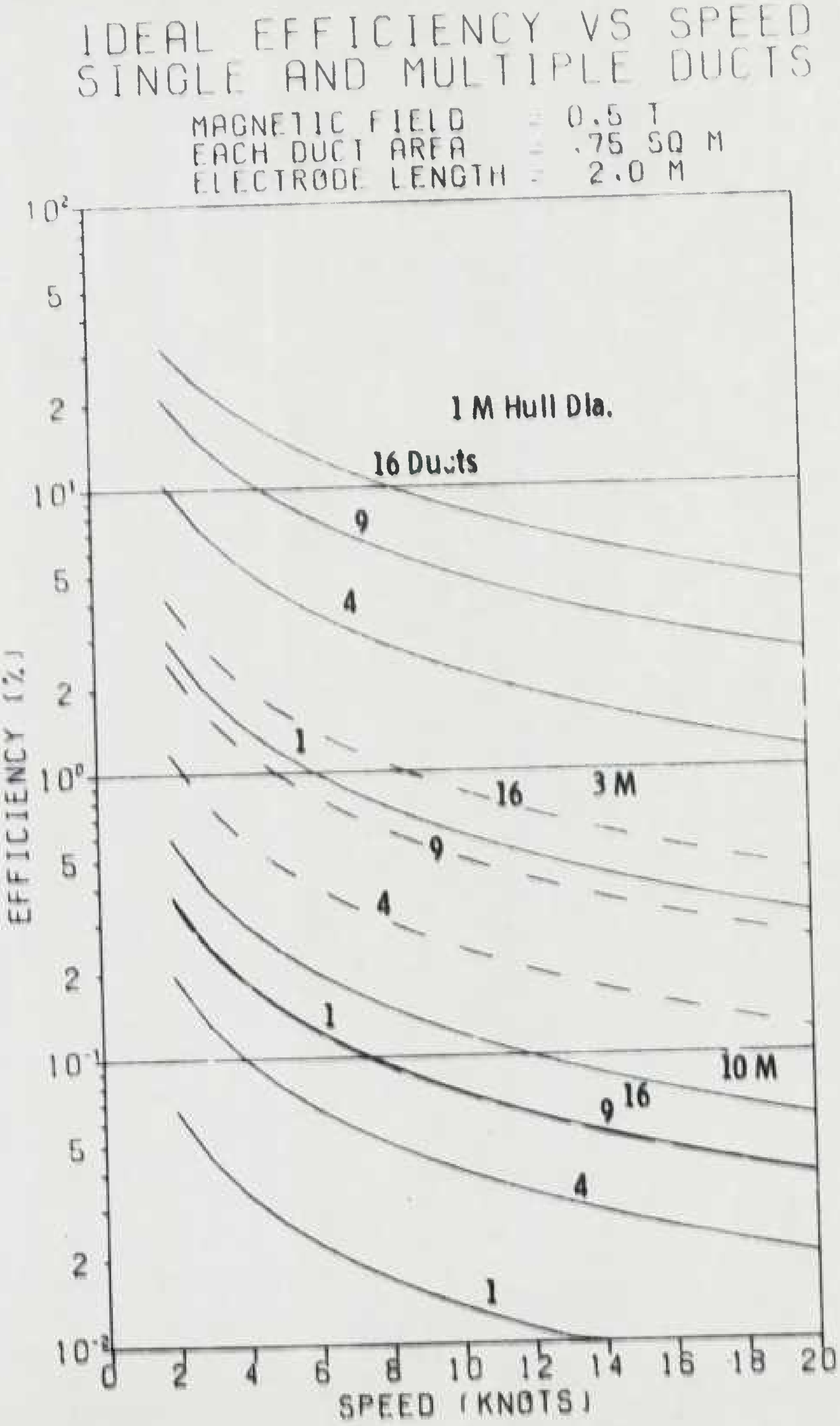
EACH DUCT AREA = .50 SQ M
ELECTRODE LENGTH = 2.0 M
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
1
10 M
16
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 5


IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
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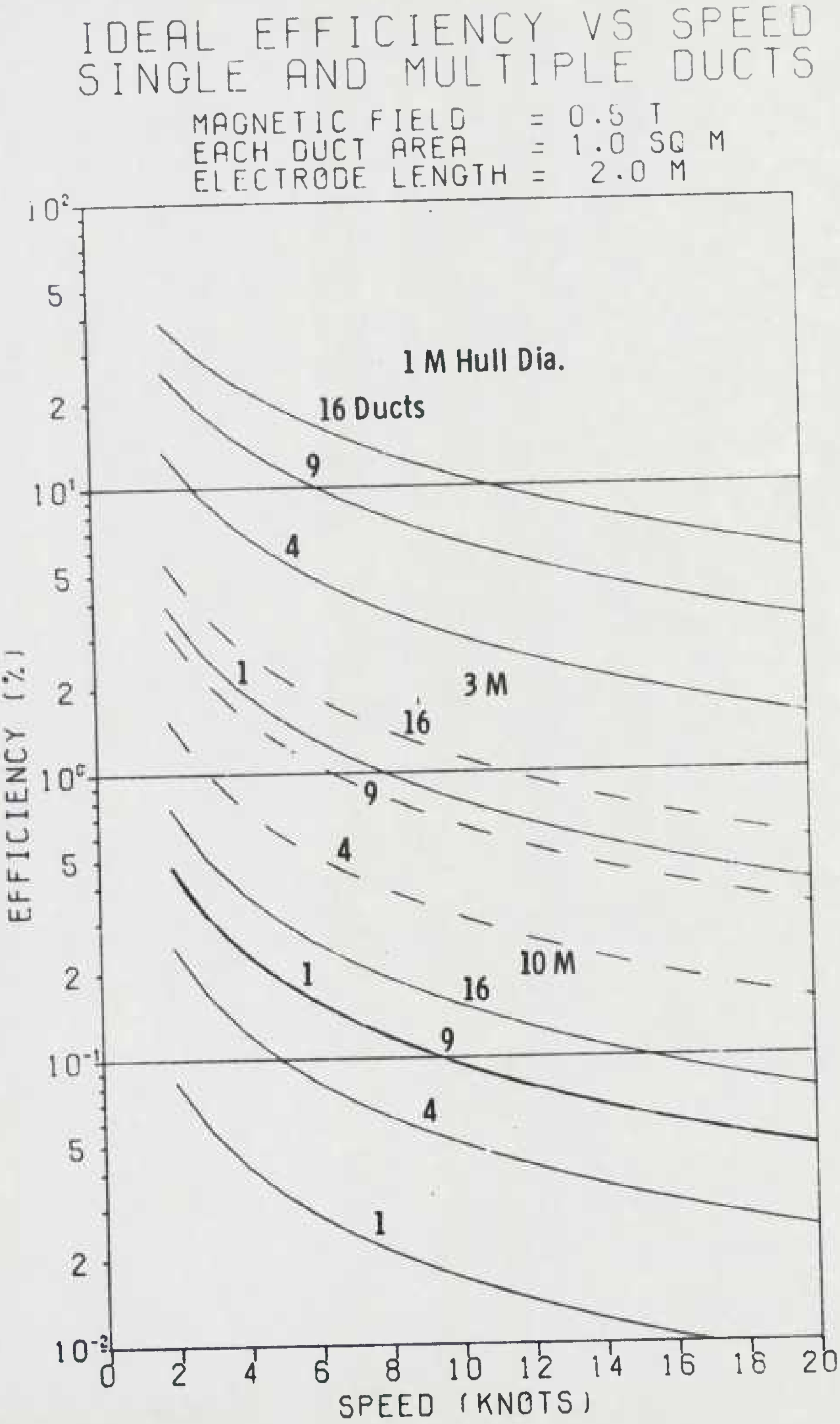
EACH DUCT AREA = .75 SQ M
ELECTRODE LENGTH = 2.0 M
1 M Hull Dia.
16 Ducts
9
4
1
16
3 M
9
4
1
10 M
9
16
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 6



IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 0.5 T
EACH DUCT AREA = 1.0 SQ M
ELECTRODE LENGTH = 2.0 M
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
10 M
1
16
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 7

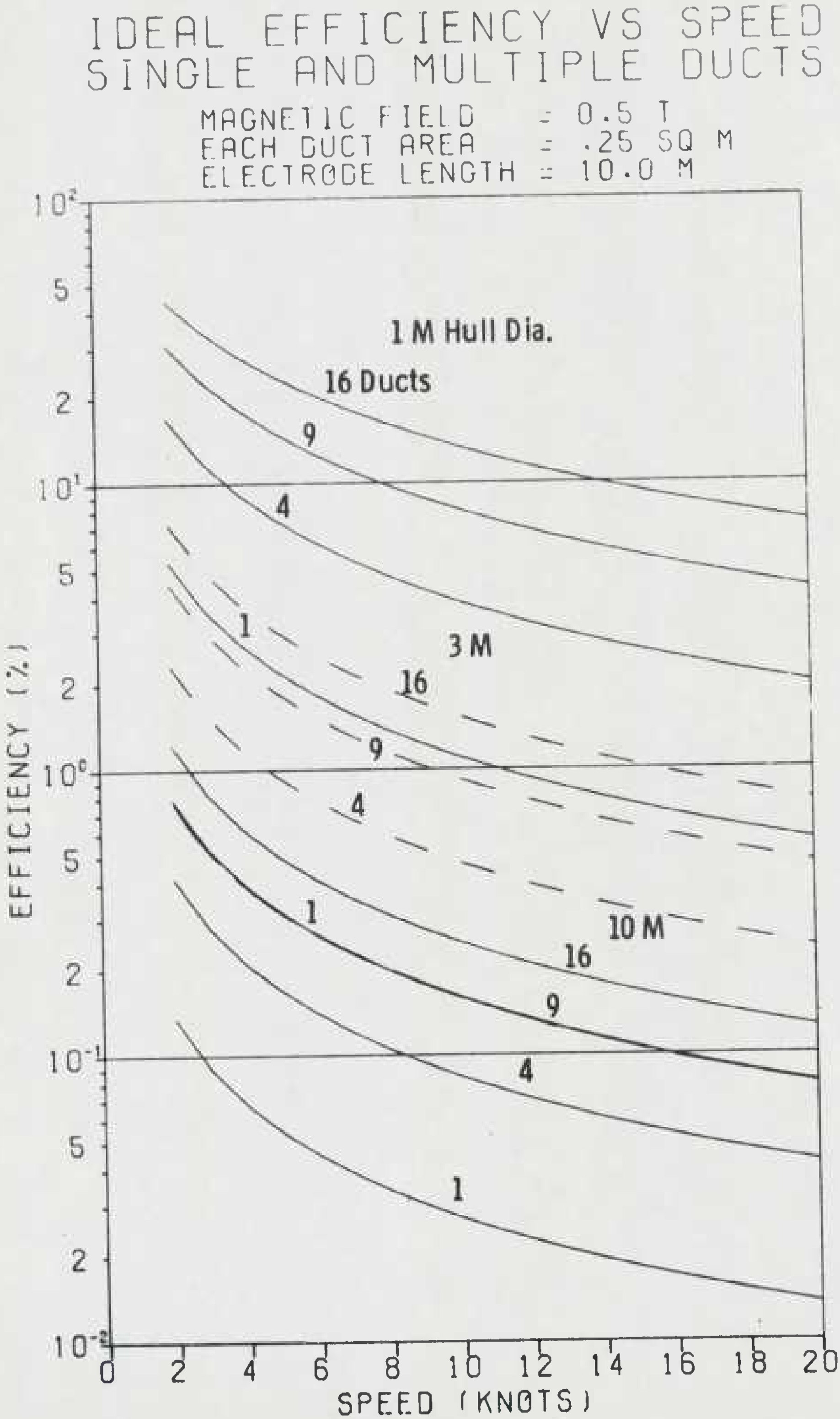
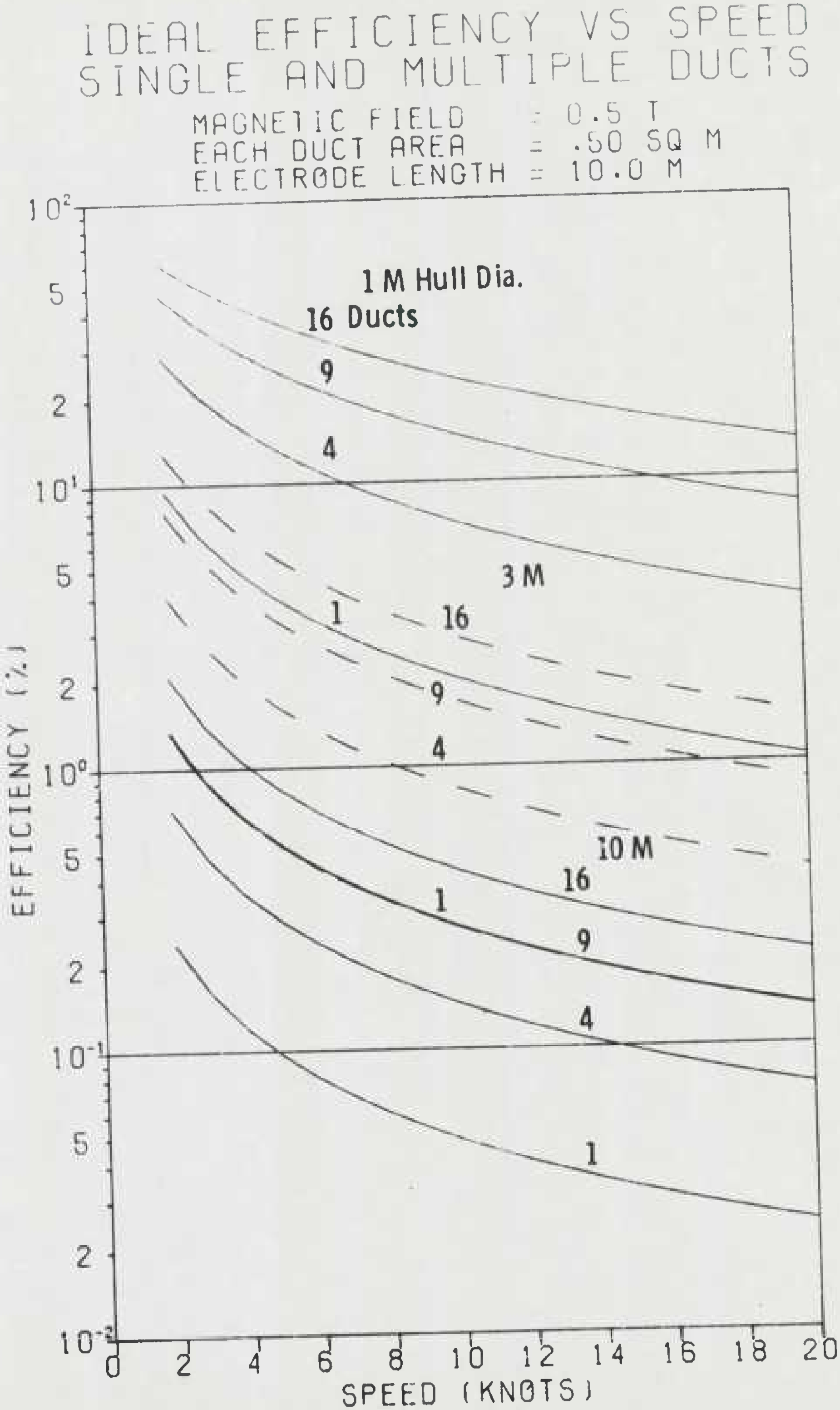

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 0.5 T
EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 10.0 M
EFFICIENCY (%)
SPEED (KNOTS)
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
1
10 M
16
9
4
1
10²
5
2
10¹
5
2
10⁰
5
2
10⁻¹
5
2
10⁻²
0
2
4
6
8
10
12
14
16
18
20

FIGURE 8


IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
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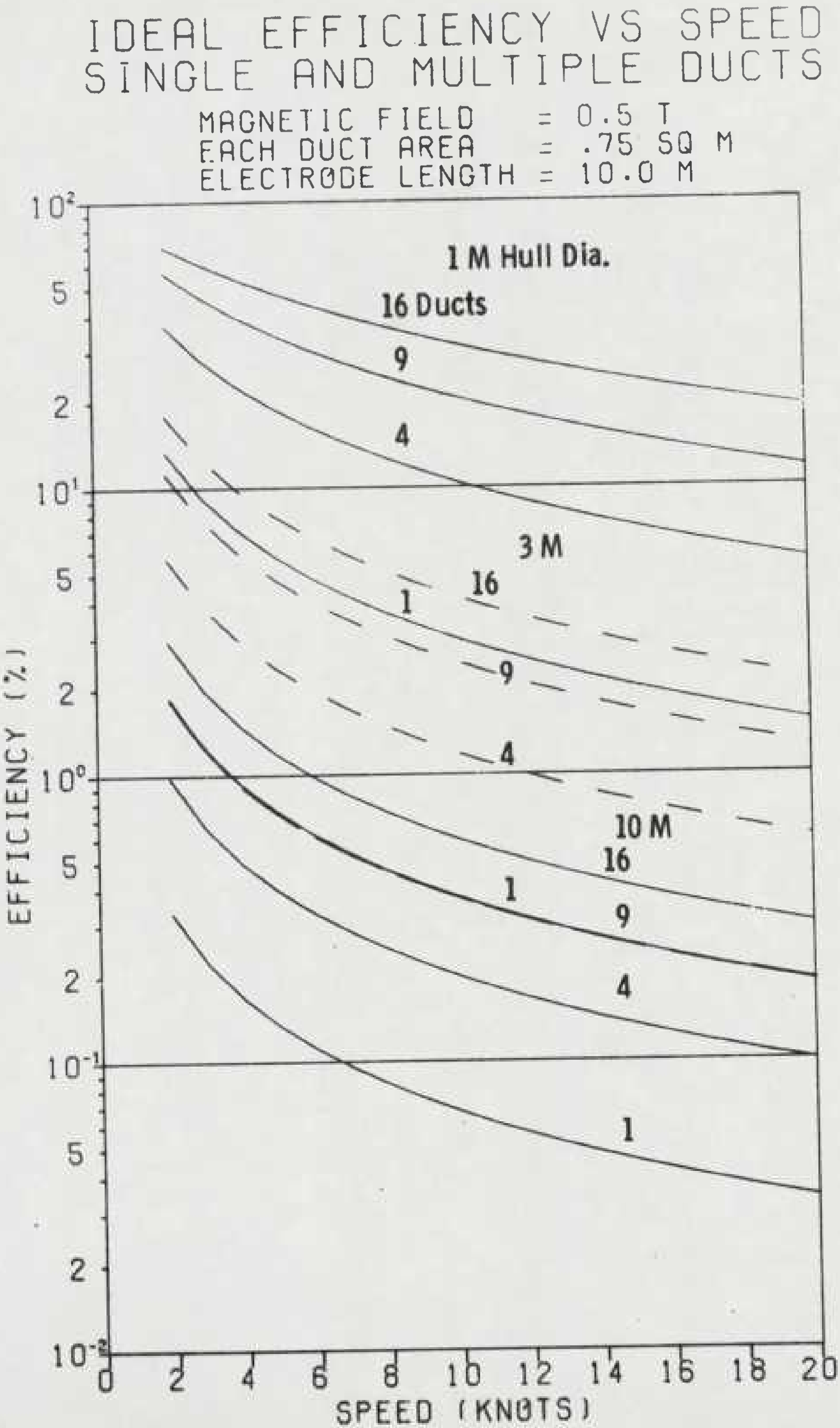
EACH DUCT AREA = .50 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
3 M
1
16
9
4
10 M
16
1
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)
10²
5
2
10¹
5
2
10⁰
5
2
10⁻¹
5
2
10⁻²
0
2
4
6
8
10
12
14
16
18
20

FIGURE 9


IDEAL EFFICIENCY VS SPEED
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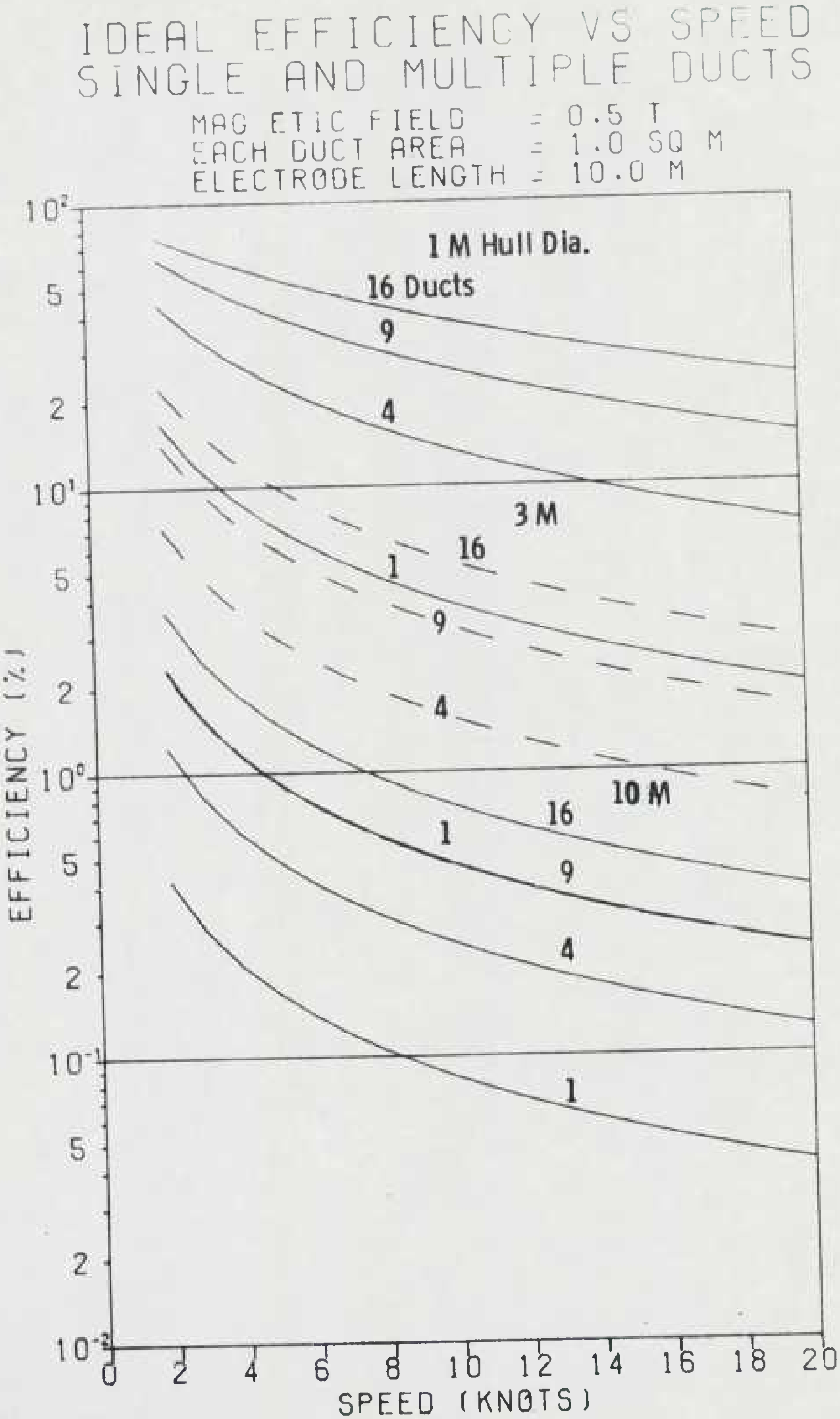
EACH DUCT AREA = .75 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
3 M
16
1
9
4
10 M
16
1
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)
0
2
4
6
8
10
12
14
16
18
20
10^{2}
5
2
10^{1}
5
2
10^{0}
5
2
10^{-1}
5
2
10^{-2}

FIGURE 10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SINGLE AND MULTIPLE DUCTS
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ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
3 M
16
1
9
4
10 M
16
1
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)
0
2
4
6
8
10
12
14
16
18
20

FIGURE 11

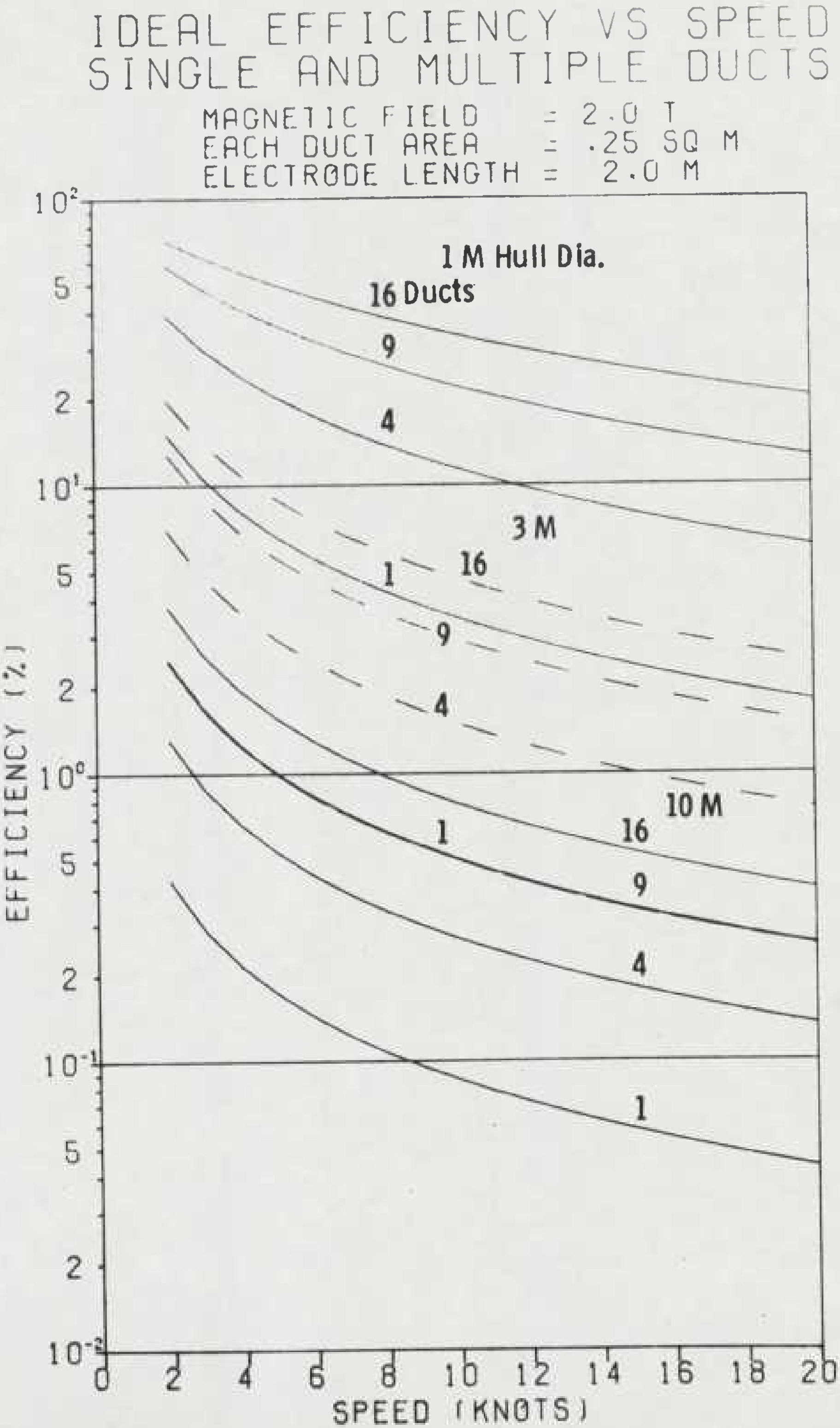
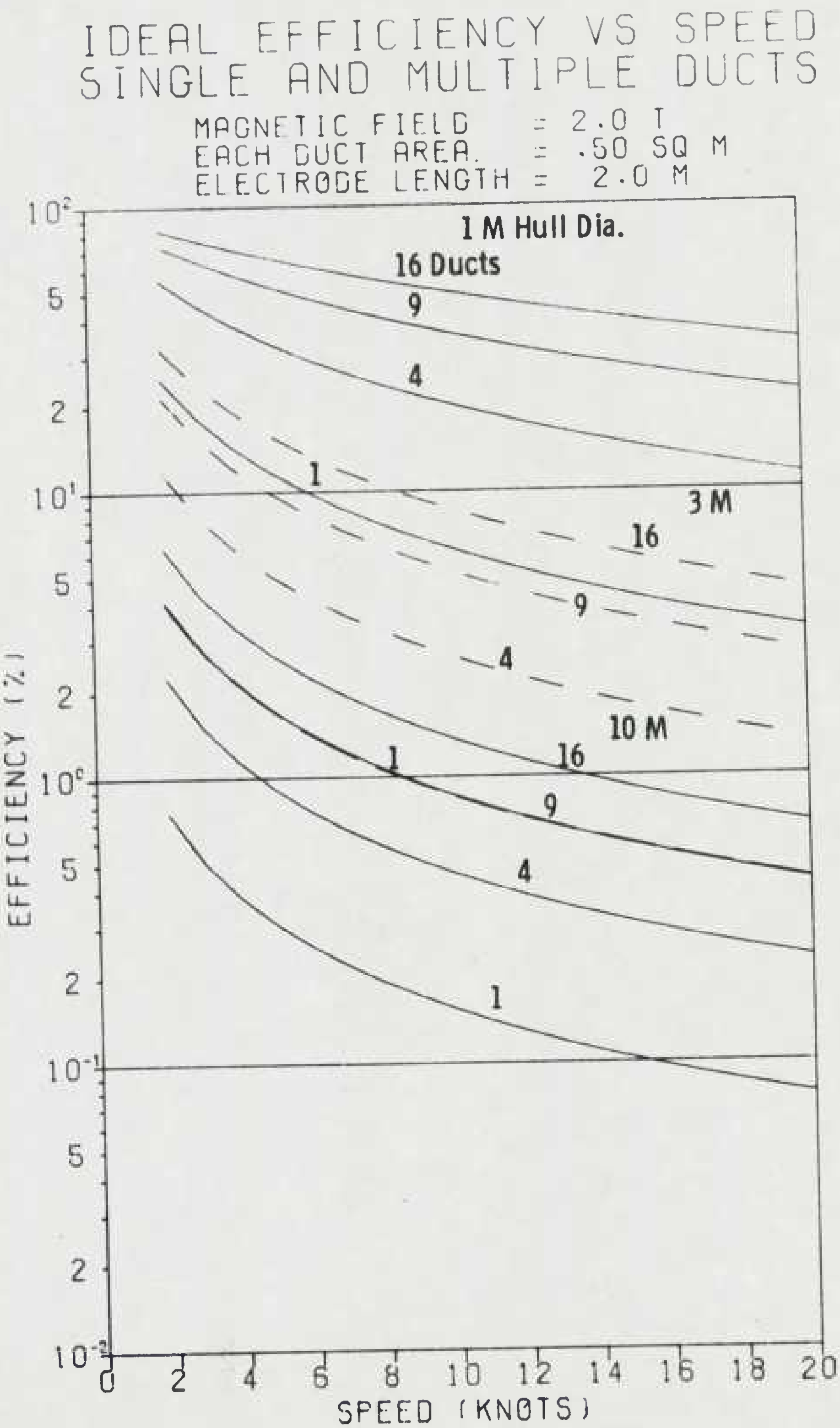

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 2.0 M
1 M Hull Dia.
16 Ducts
9
4
3 M
16
1
9
4
10 M
16
1
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)
10²
5
2
10¹
10⁰
10⁻¹
10⁻²
0
2
4
6
8
10
12
14
16
18
20

FIGURE 12


IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
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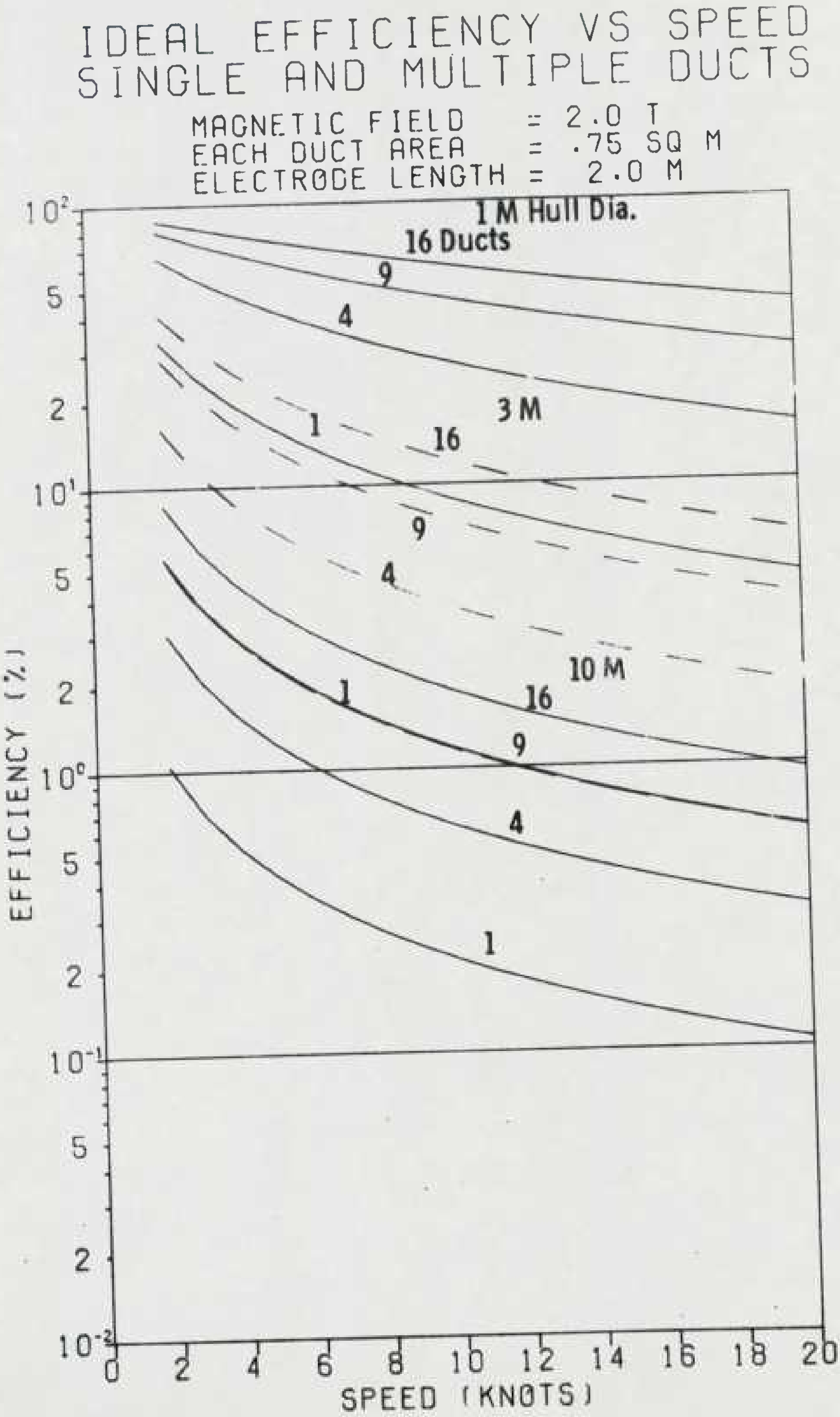
EACH DUCT AREA. = .50 SQ M
ELECTRODE LENGTH = 2.0 M
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
10 M
1
16
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)
10²
5
2
10¹
10⁰
10⁻¹
10⁻²
0
2
4
6
8
10
12
14
16
18
20

FIGURE 13



IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
EACH DUCT AREA = .75 SQ M
ELECTRODE LENGTH = 2.0 M
1 M Hull Dia.
16 Ducts
9
4
3 M
1
16
9
4
10 M
16
1
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)
10²
5
2
10¹
5
2
10⁰
5
2
10⁻¹
5
2
10⁻²
0
2
4
6
8
10
12
14
16
18
20

FIGURE 14

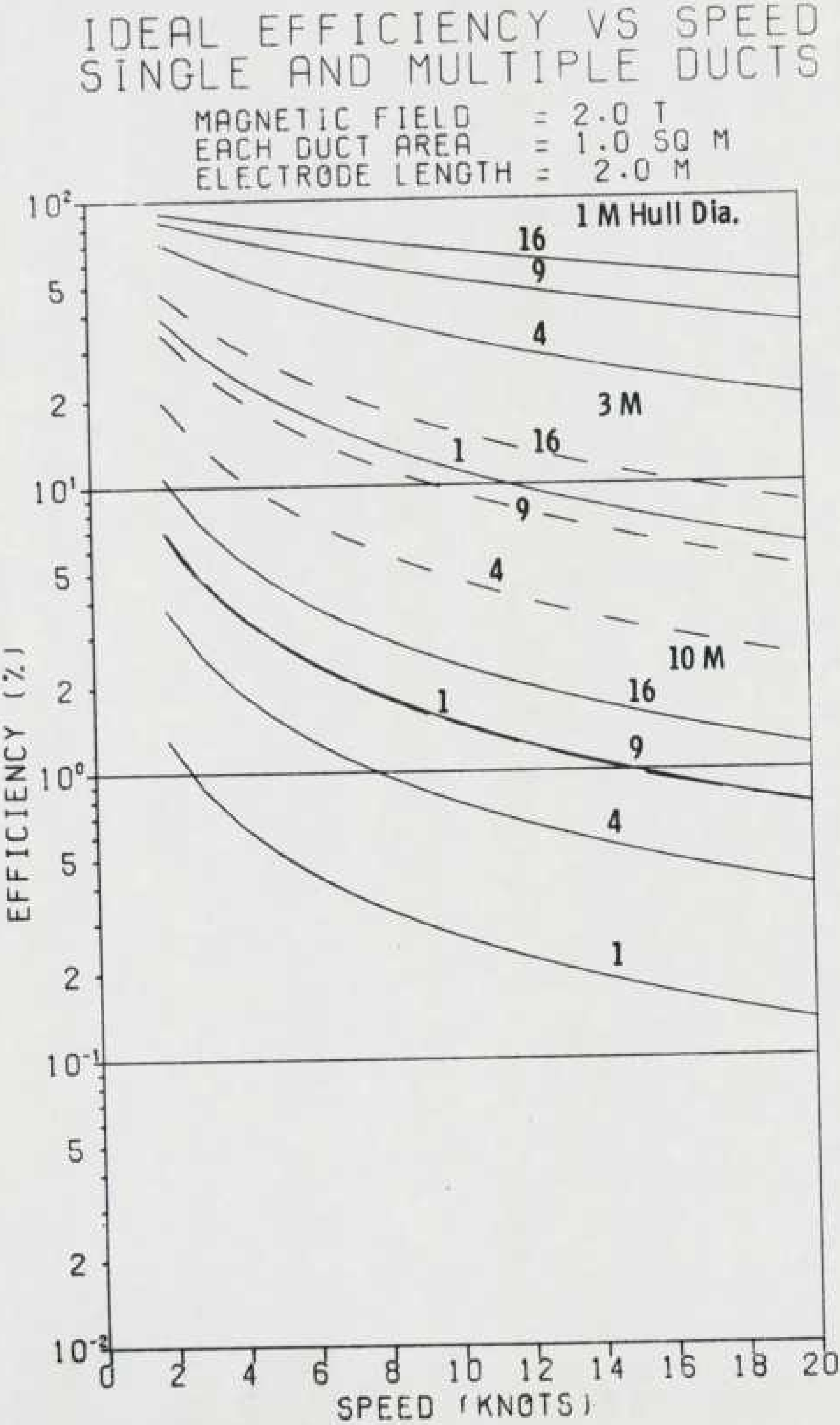

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
EACH DUCT AREA = 1.0 SQ M
ELECTRODE LENGTH = 2.0 M
1 M Hull Dia.
16
9
4
3 M
16
1
9
4
10 M
16
1
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)
0
2
4
6
8
10
12
14
16
18
20

FIGURE 15

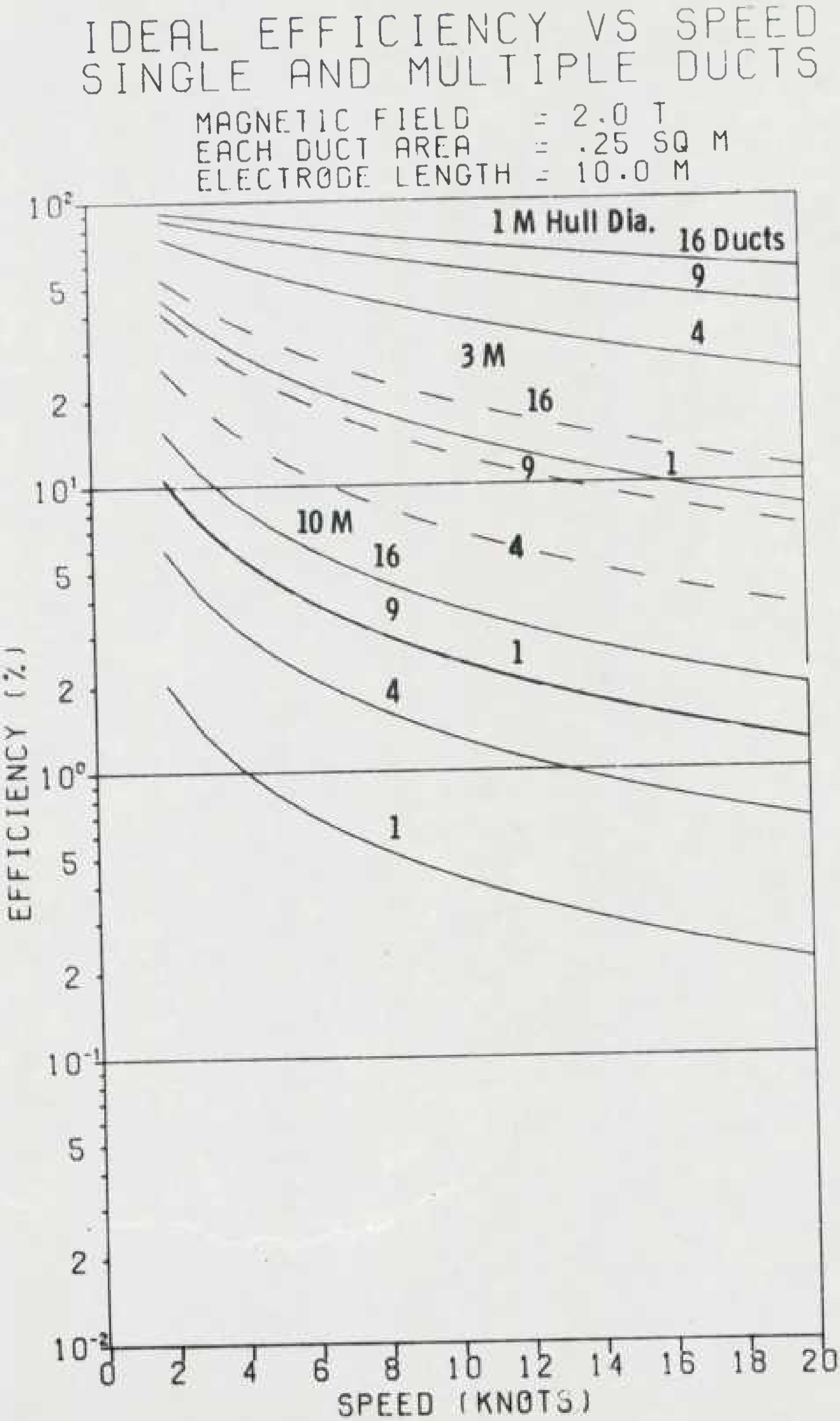
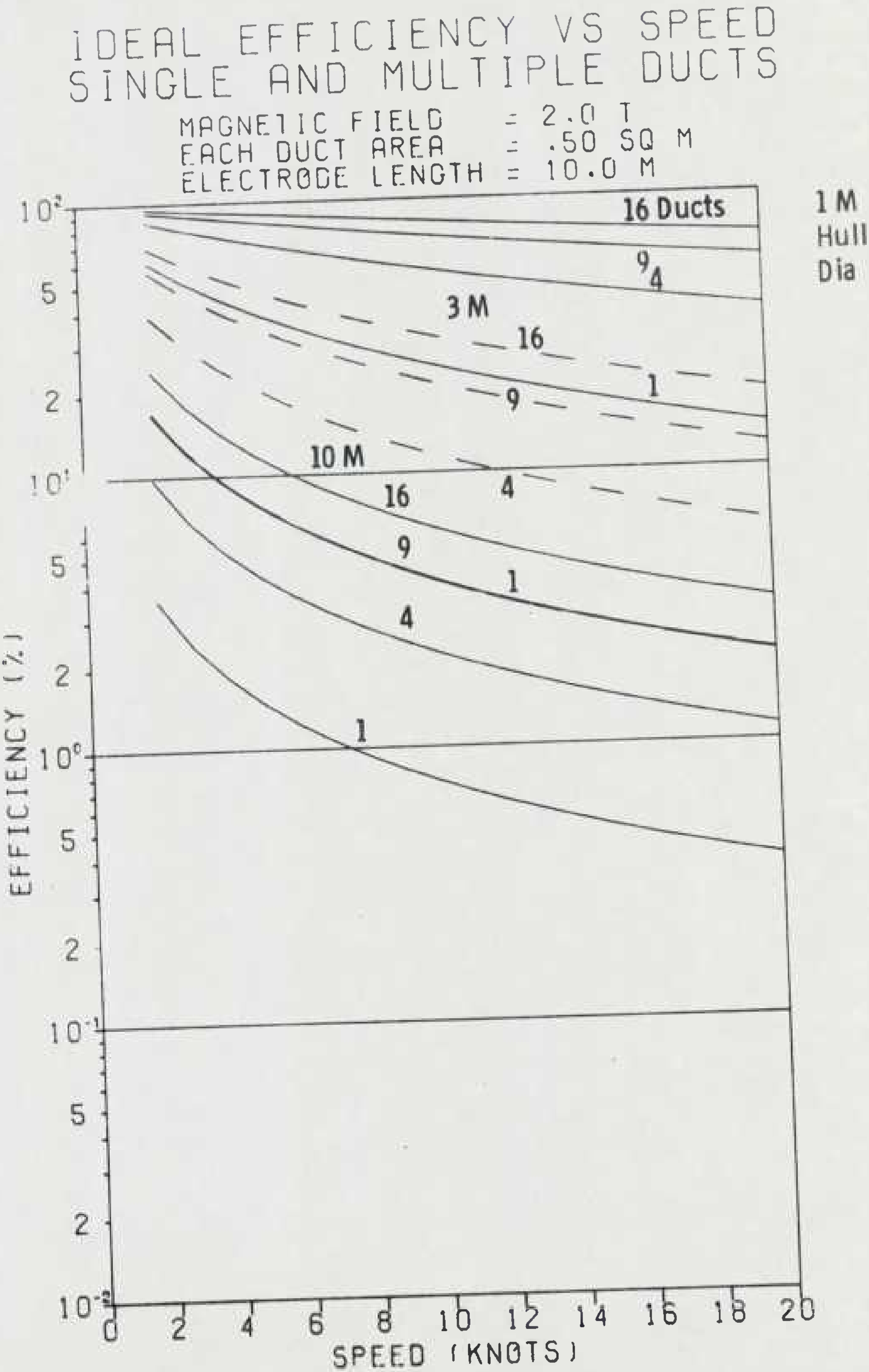

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
3 M
16
9
1
4
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 16



IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
EACH DUCT AREA = .50 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia
16 Ducts
9
4
3 M
16
1
9
10 M
4
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 17

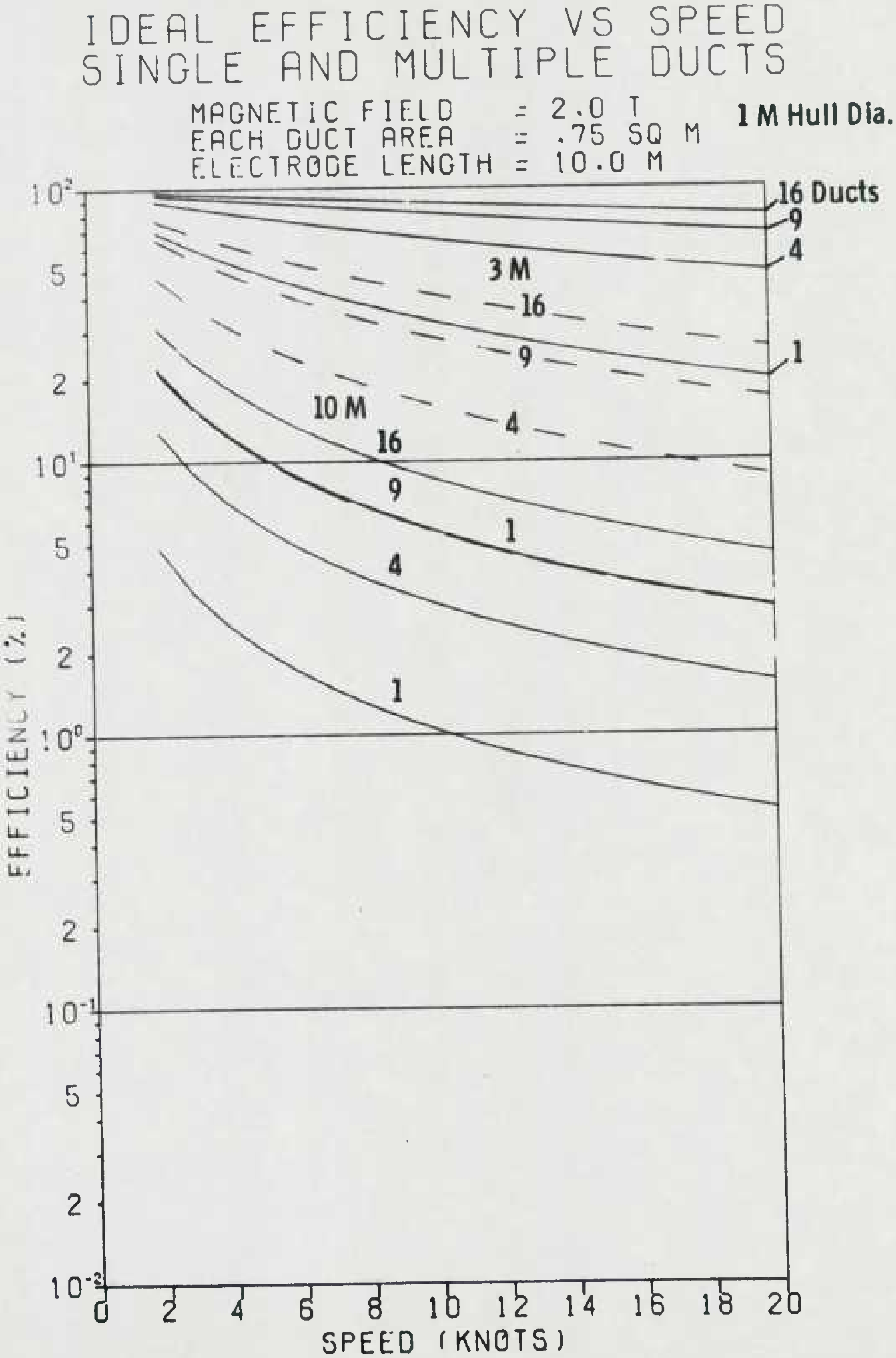

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
EACH DUCT AREA = .75 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
3 M
16
9
1
4
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 18

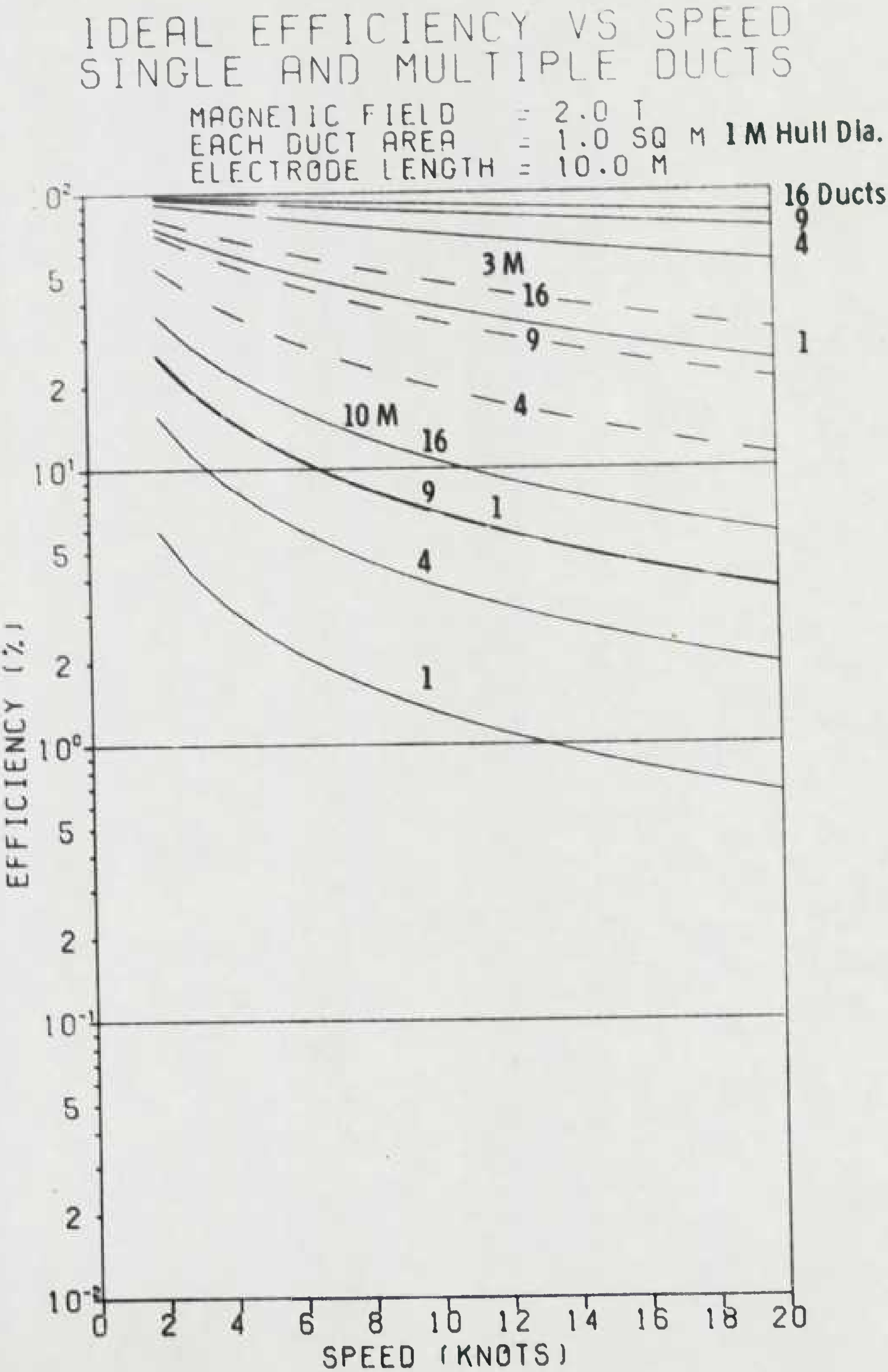

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
EACH DUCT AREA = 1.0 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
1
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)
0
2
4
6
8
10
12
14
16
18
20

FIGURE 19

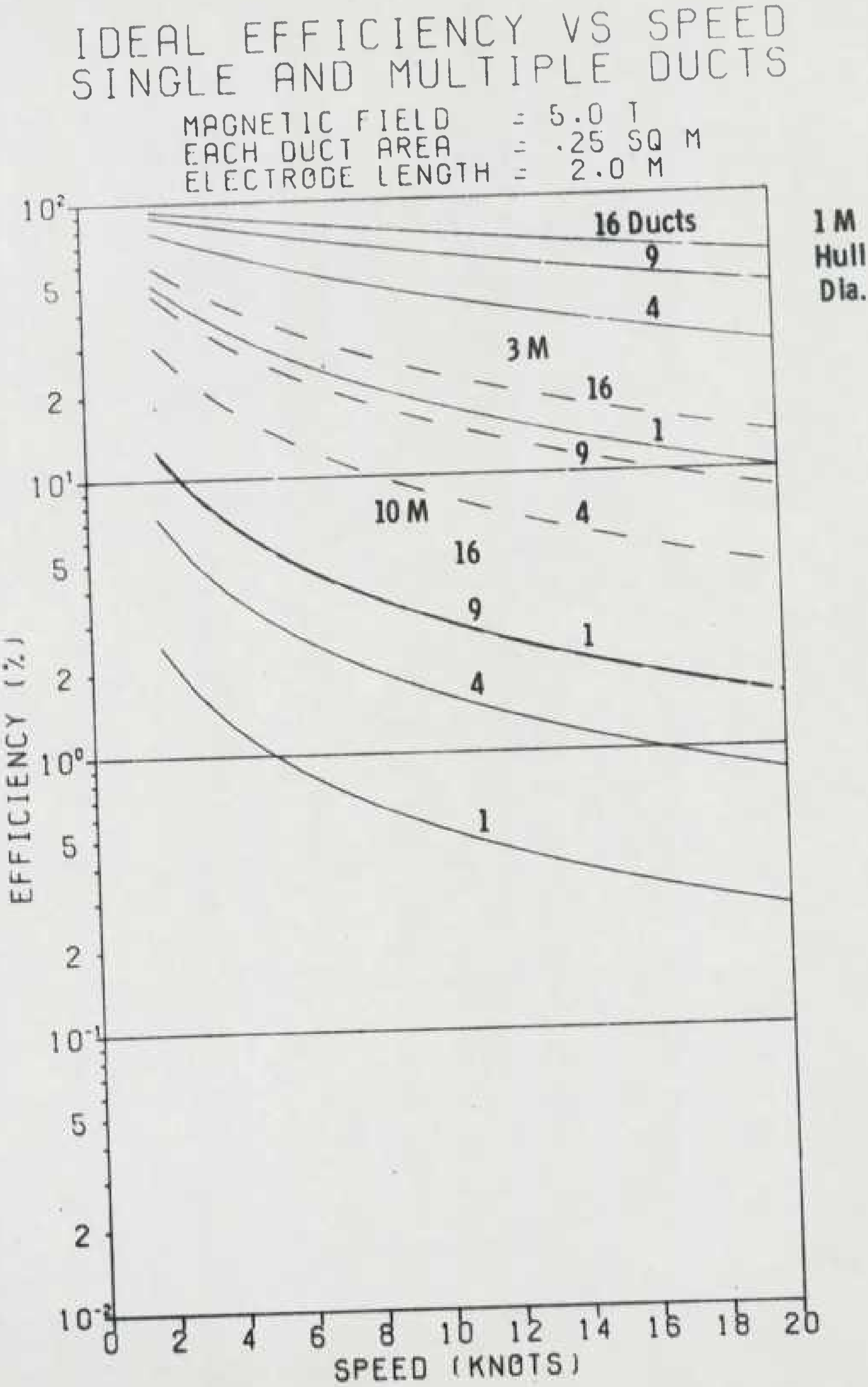

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 2.0 M
16 Ducts
9
4
1 M
Hull
Dia.
3 M
16
1
9
4
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 20

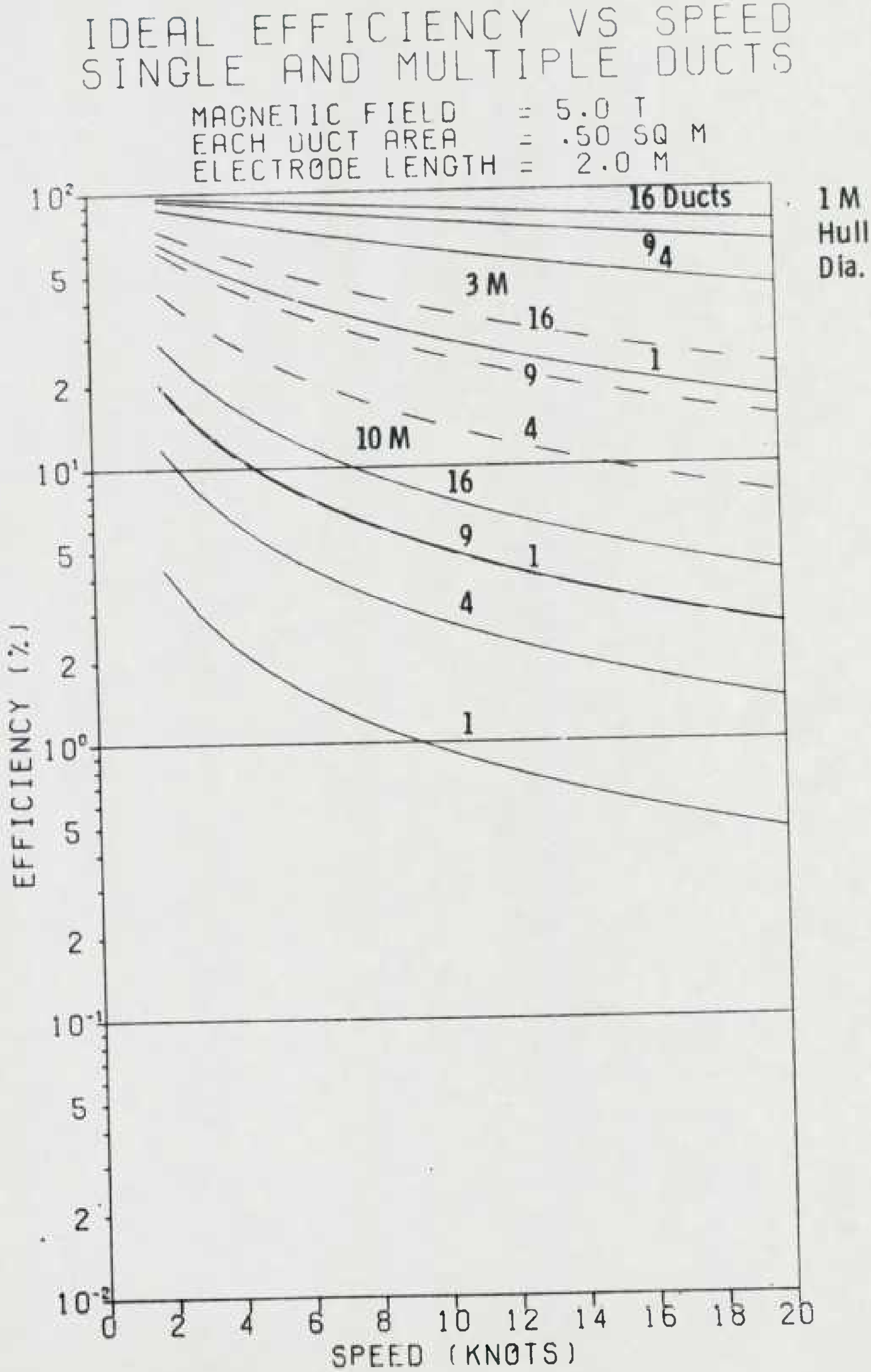
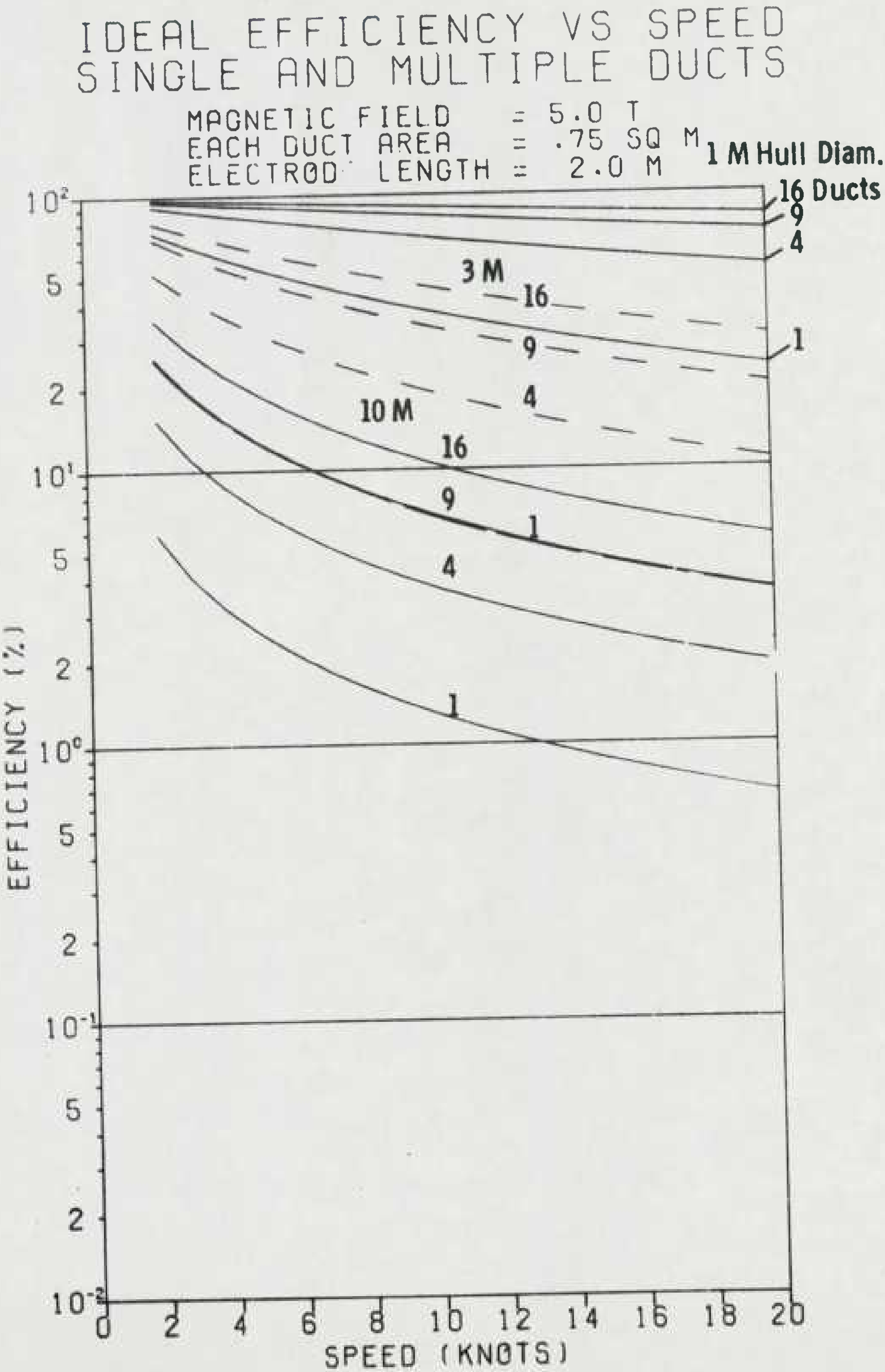
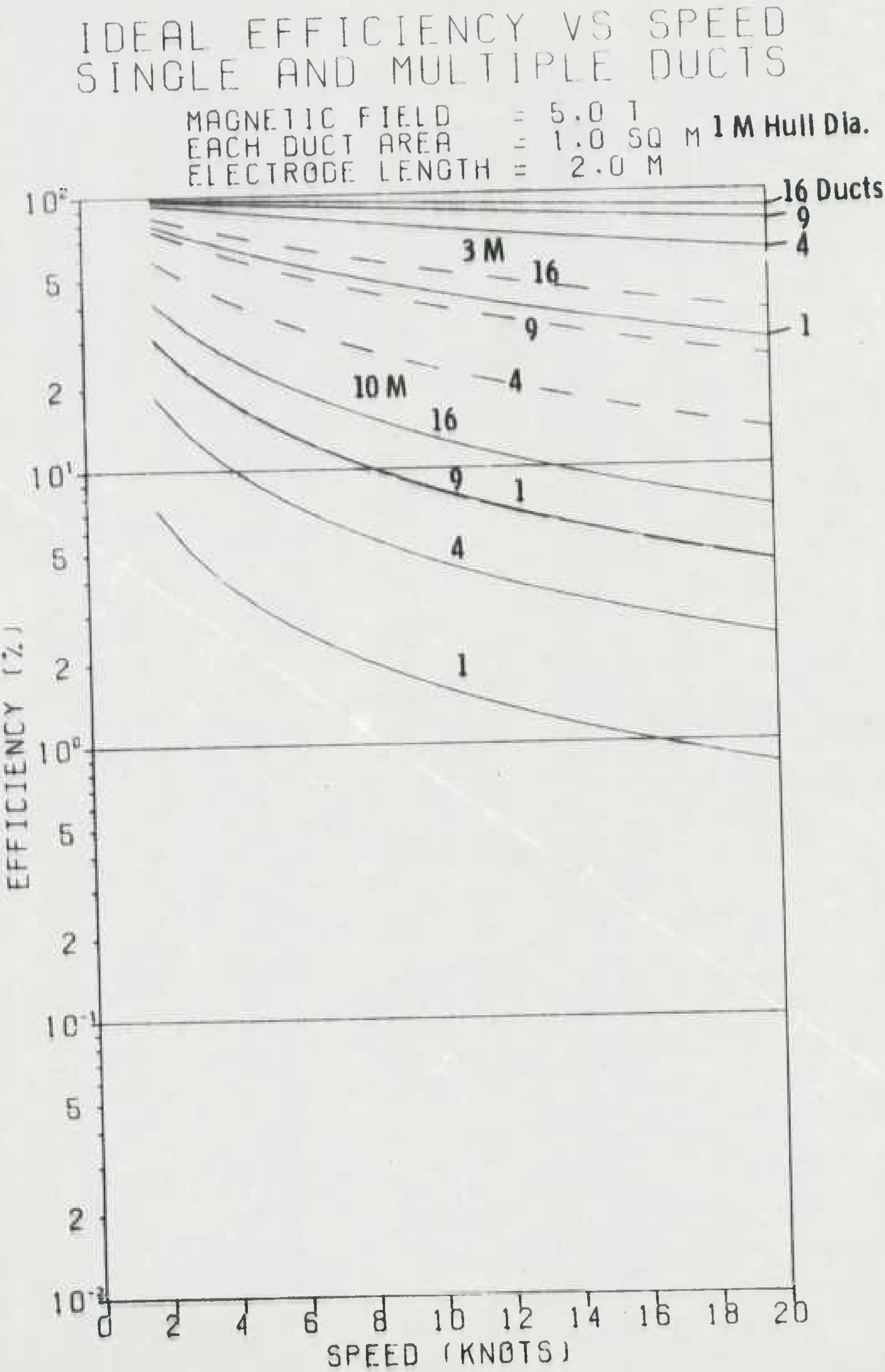

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = .50 SQ M
ELECTRODE LENGTH = 2.0 M
16 Ducts
9
4
1 M Hull Dia.
3 M
16
9
1
4
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 21



IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = .75 SQ M
ELECTROD LENGTH = 2.0 M
1 M Hull Diam.
16 Ducts
9
4
1
3 M
16
9
4
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 22



IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = 1.0 SQ M
ELECTRODE LENGTH = 2.0 M
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 23

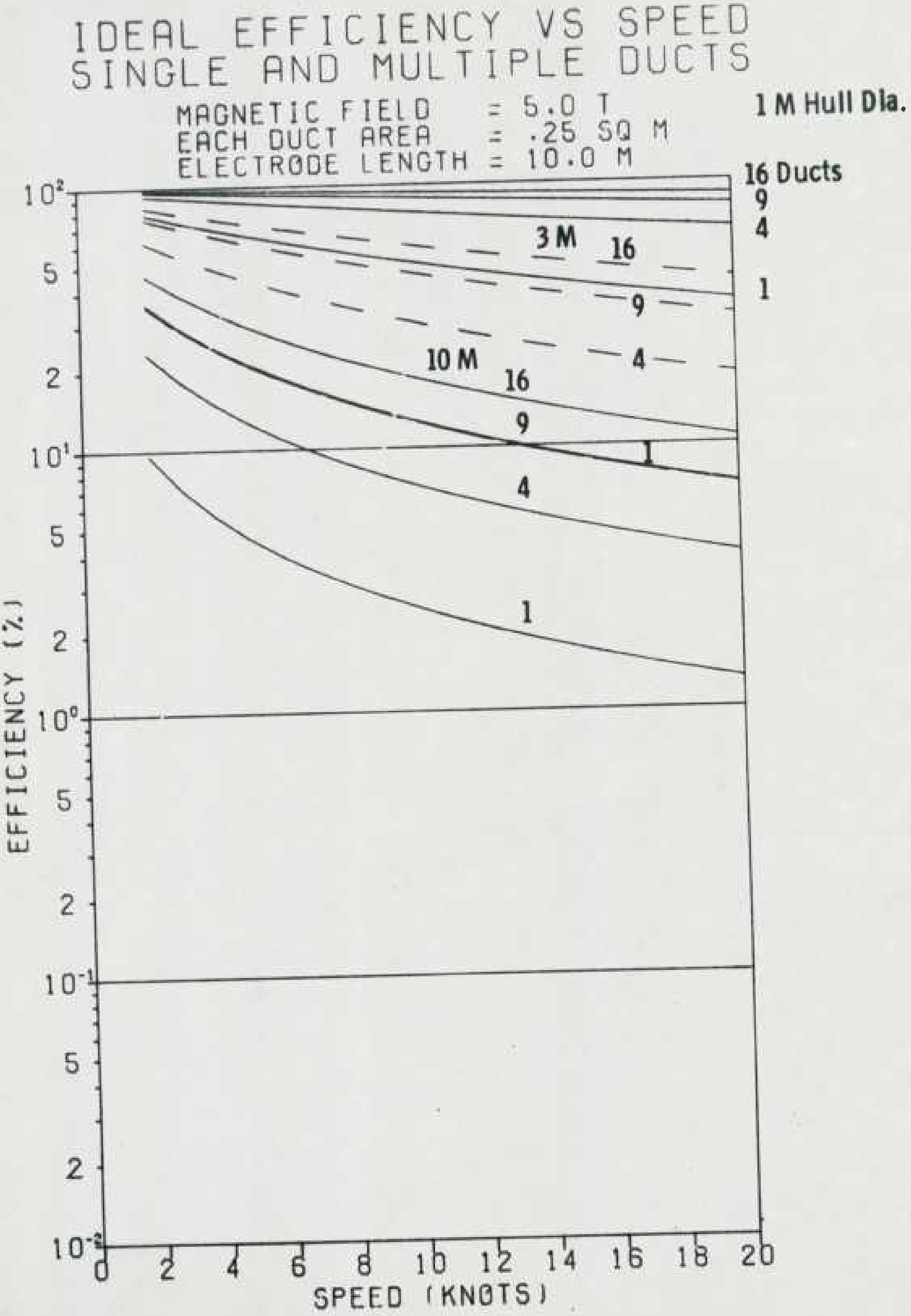

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
10 M
16
9
4
1
EFFICIENCY (%)
SPEED (KNOTS)
10^2
5
2
10^1
5
2
10^0
5
2
10^{-1}
5
2
10^{-2}
0
2
4
6
8
10
12
14
16
18
20

FIGURE 24

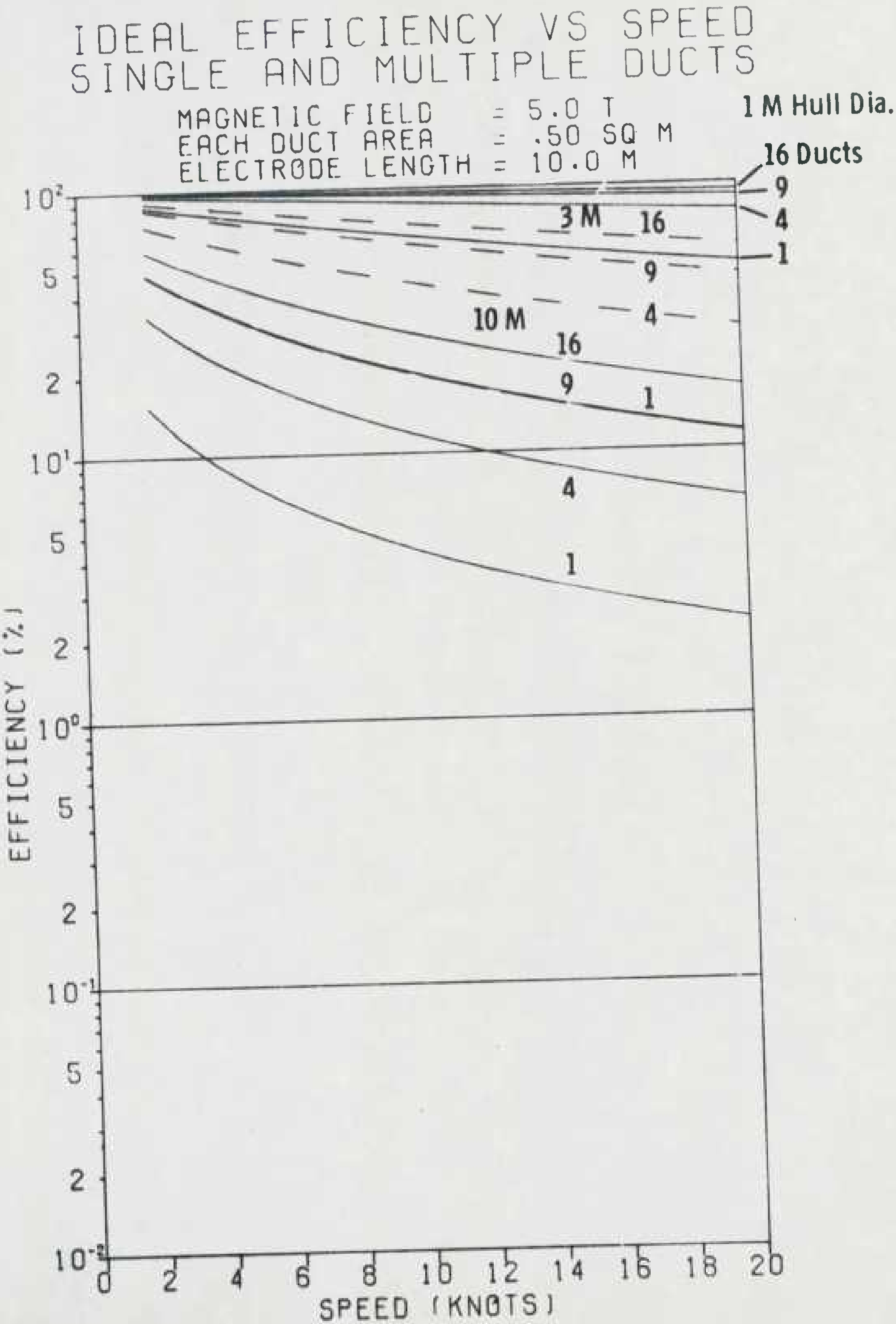

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = .50 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)
0
2
4
6
8
10
12
14
16
18
20

FIGURE 25

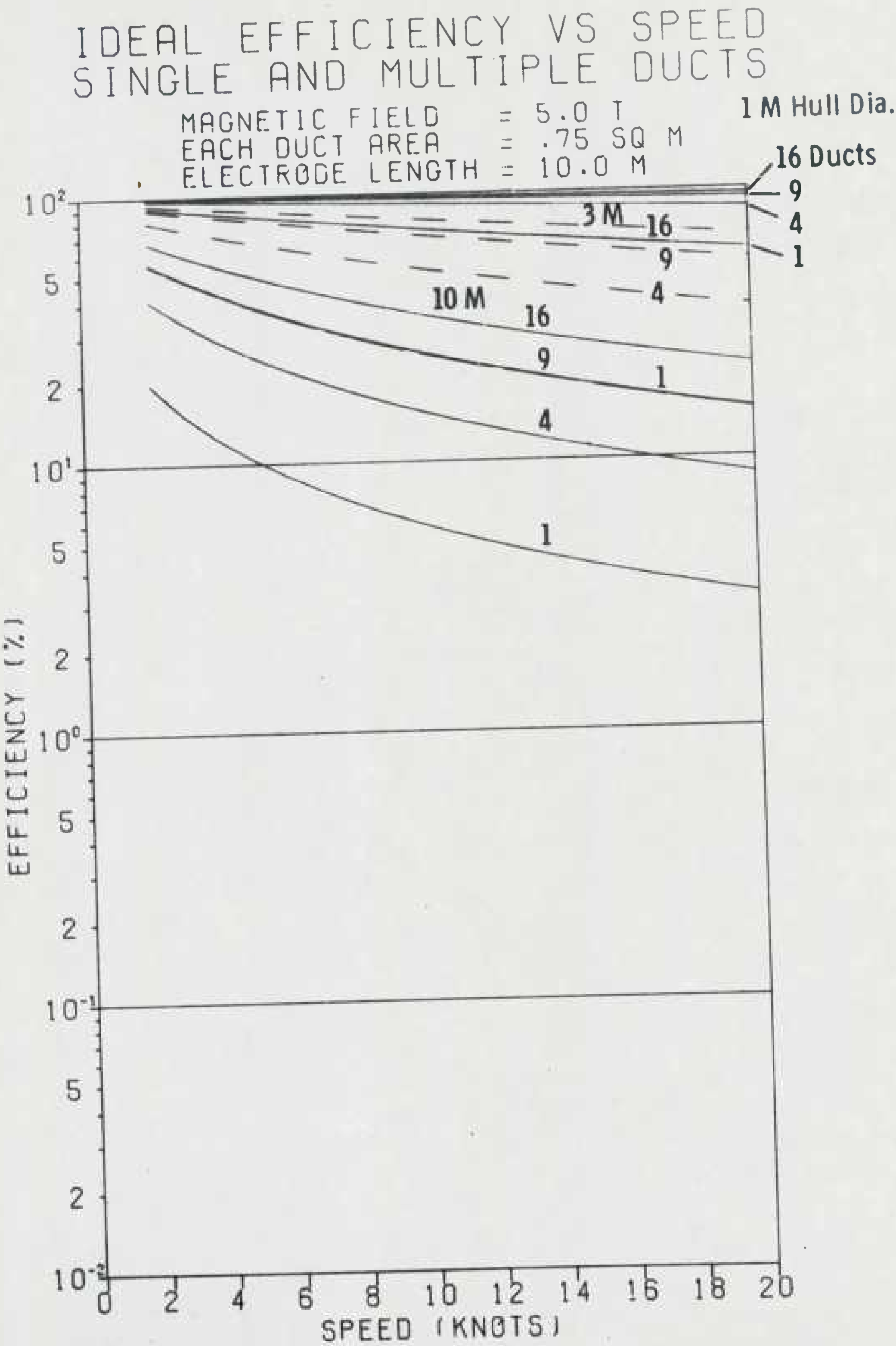
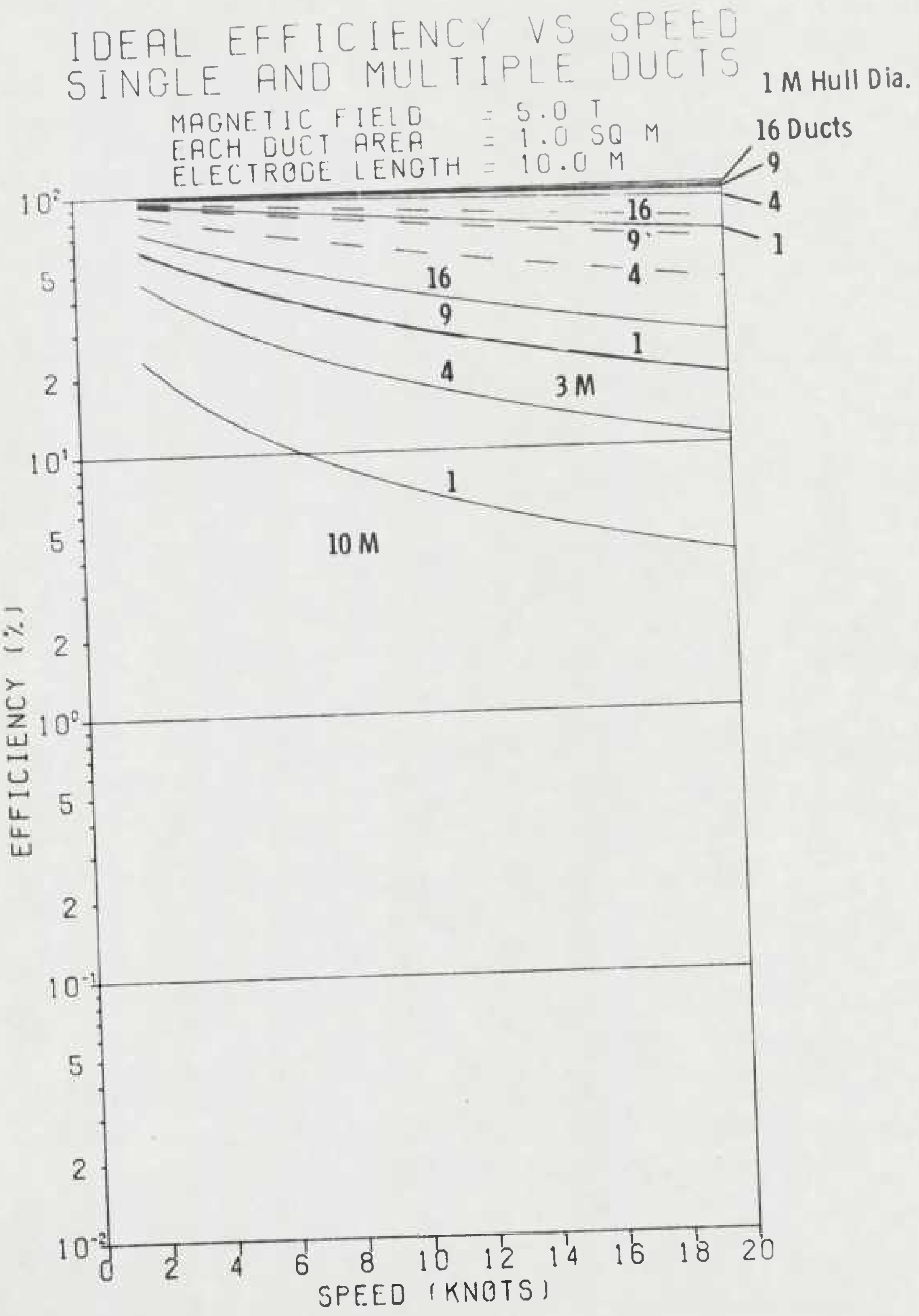

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = .75 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
1
3 M
16
9
4
10 M
16
9
1
4
1
EFFICIENCY (%)
SPEED (KNOTS)

FIGURE 26



IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = 1.0 SQ M
ELECTRODE LENGTH = 10.0 M
1 M Hull Dia.
16 Ducts
9
4
1
16
9
4
16
9
1
4
3 M
1
10 M
EFFICIENCY (%)
SPEED (KNOTS)

Curve 717915-A

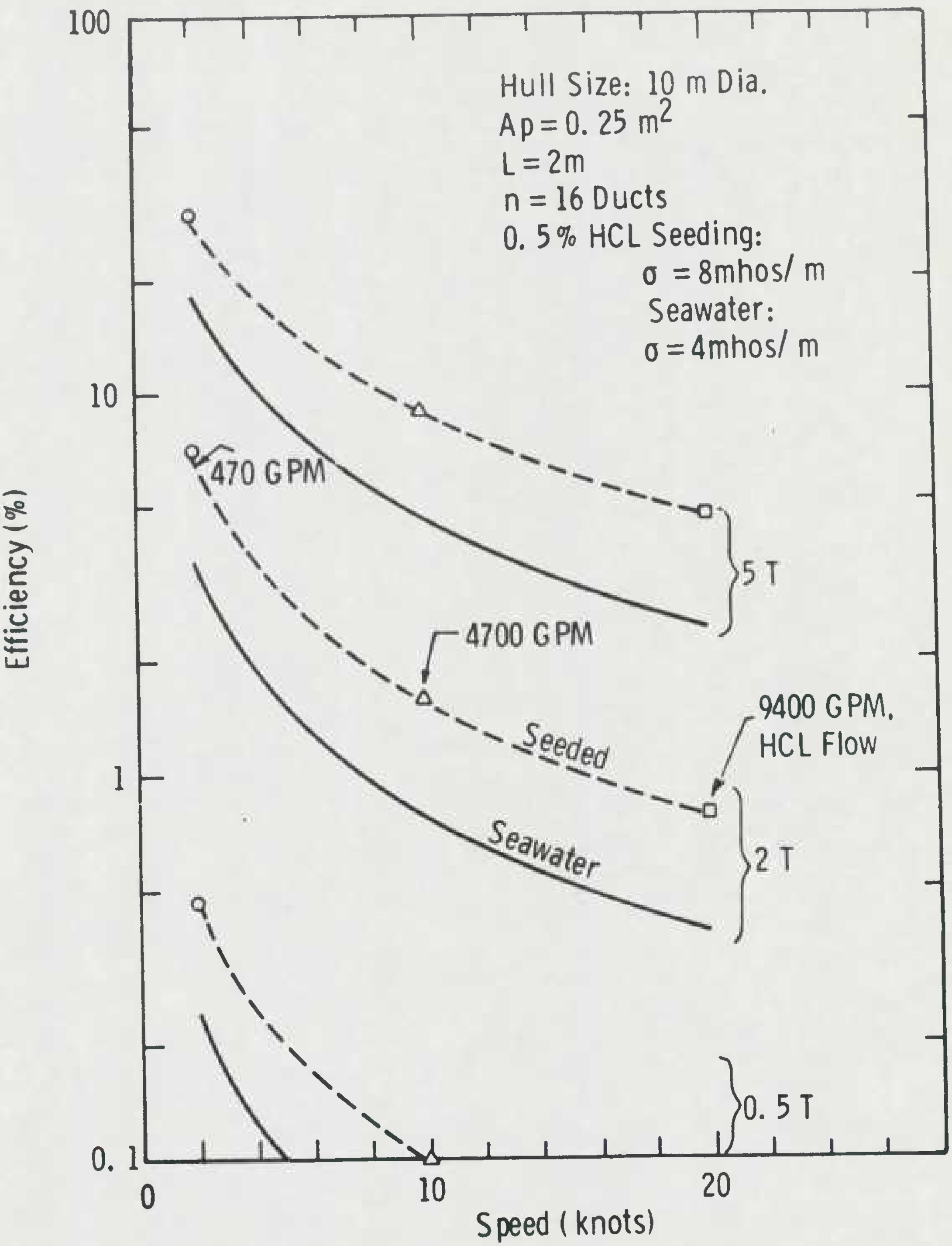


Fig. 27 — Efficiency improvement via HCL seeding: full size hull

Curve 717917-A

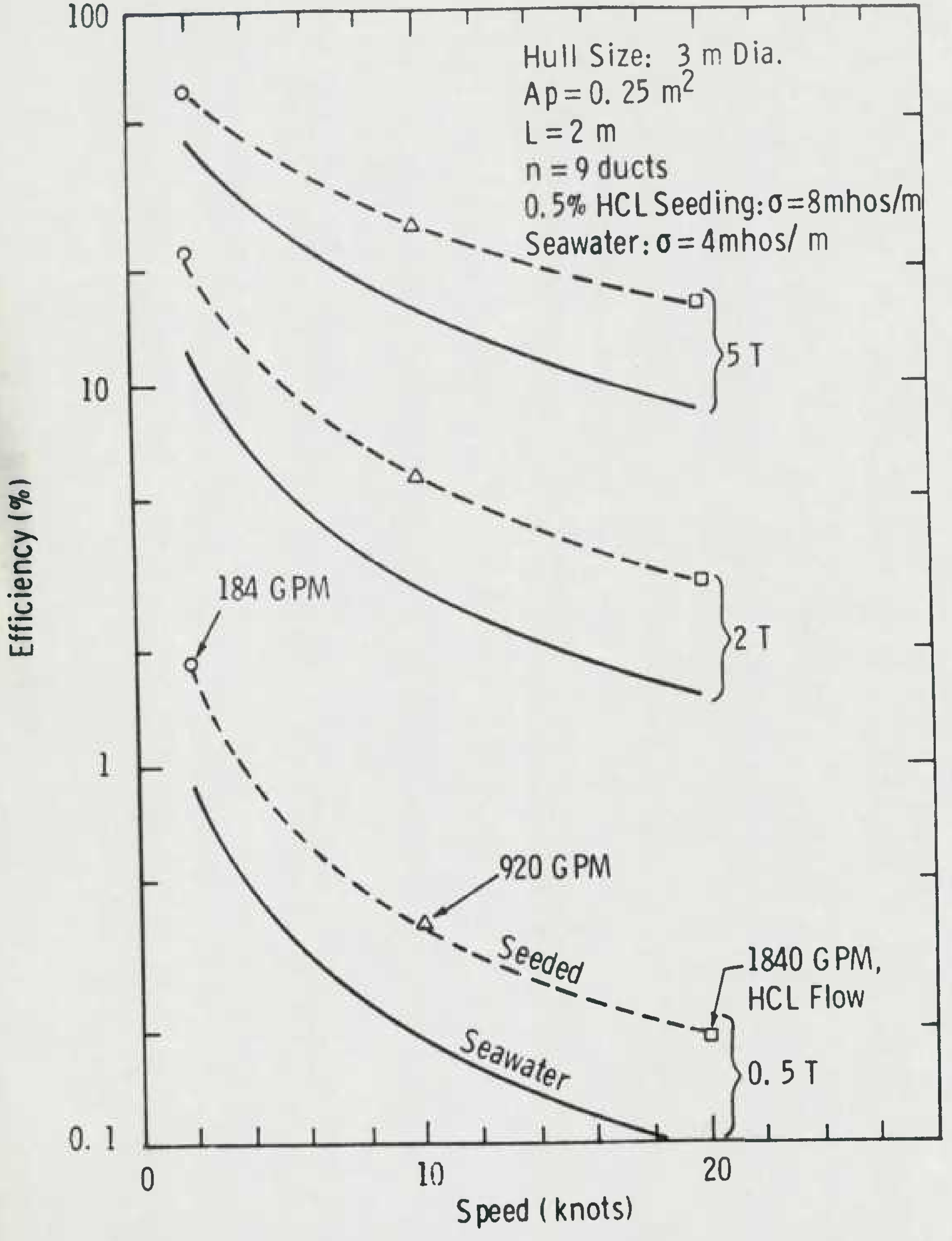


Fig. 28 — Efficiency improvement via HCL seeding: third-size hull

Curve 717916-A

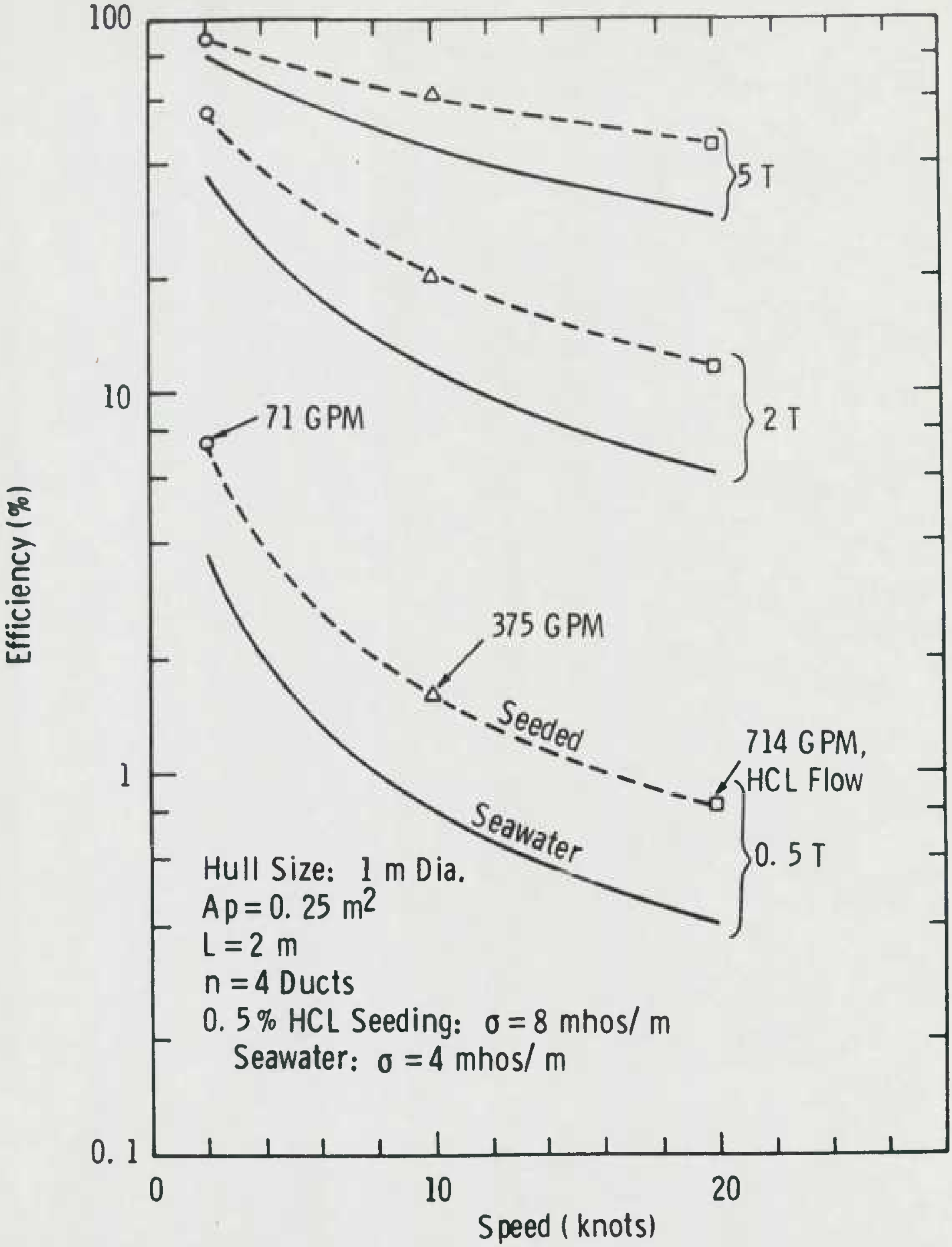


Fig. 29 — Efficiency improvement via HCL seeding: tenth-size hull

Curve 717913-A

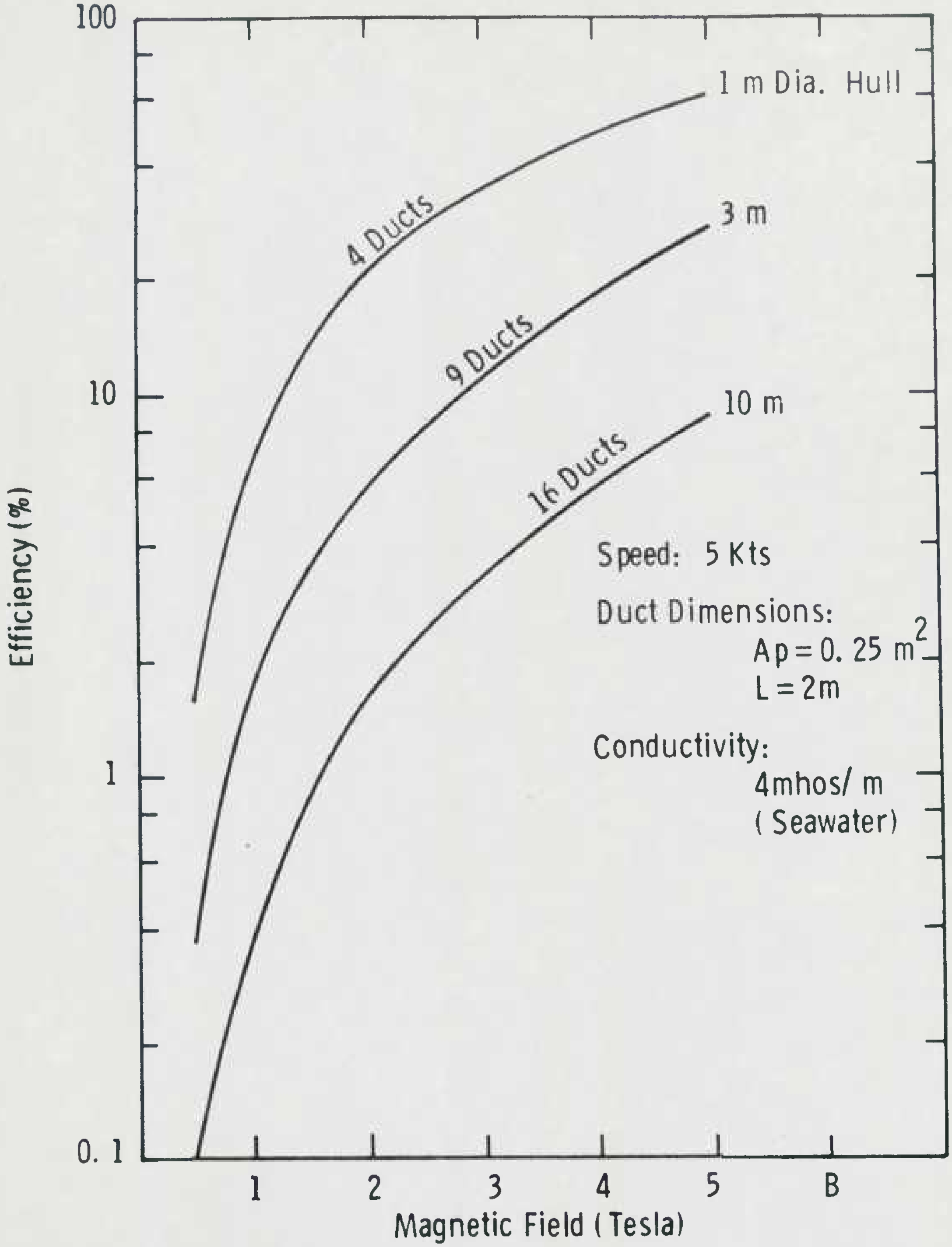


Fig. 30 — Ideal efficiency vs. magnetic field intensity for selected number of ducts & hull sizes

Curve 717914-A

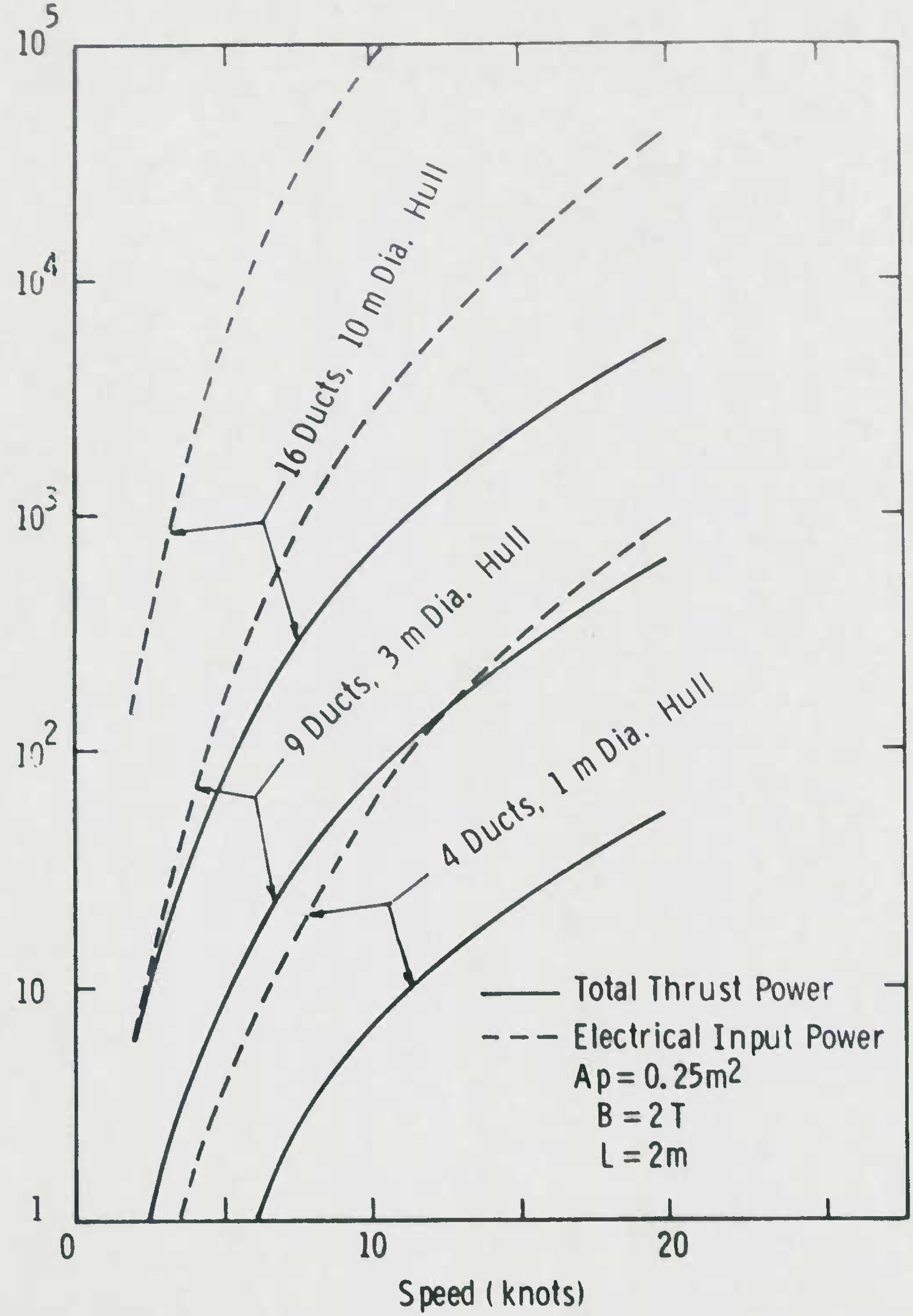


Fig. 31 — Power requirements vs. speed for selected number of ducts and hull sizes

Curve 717919-A

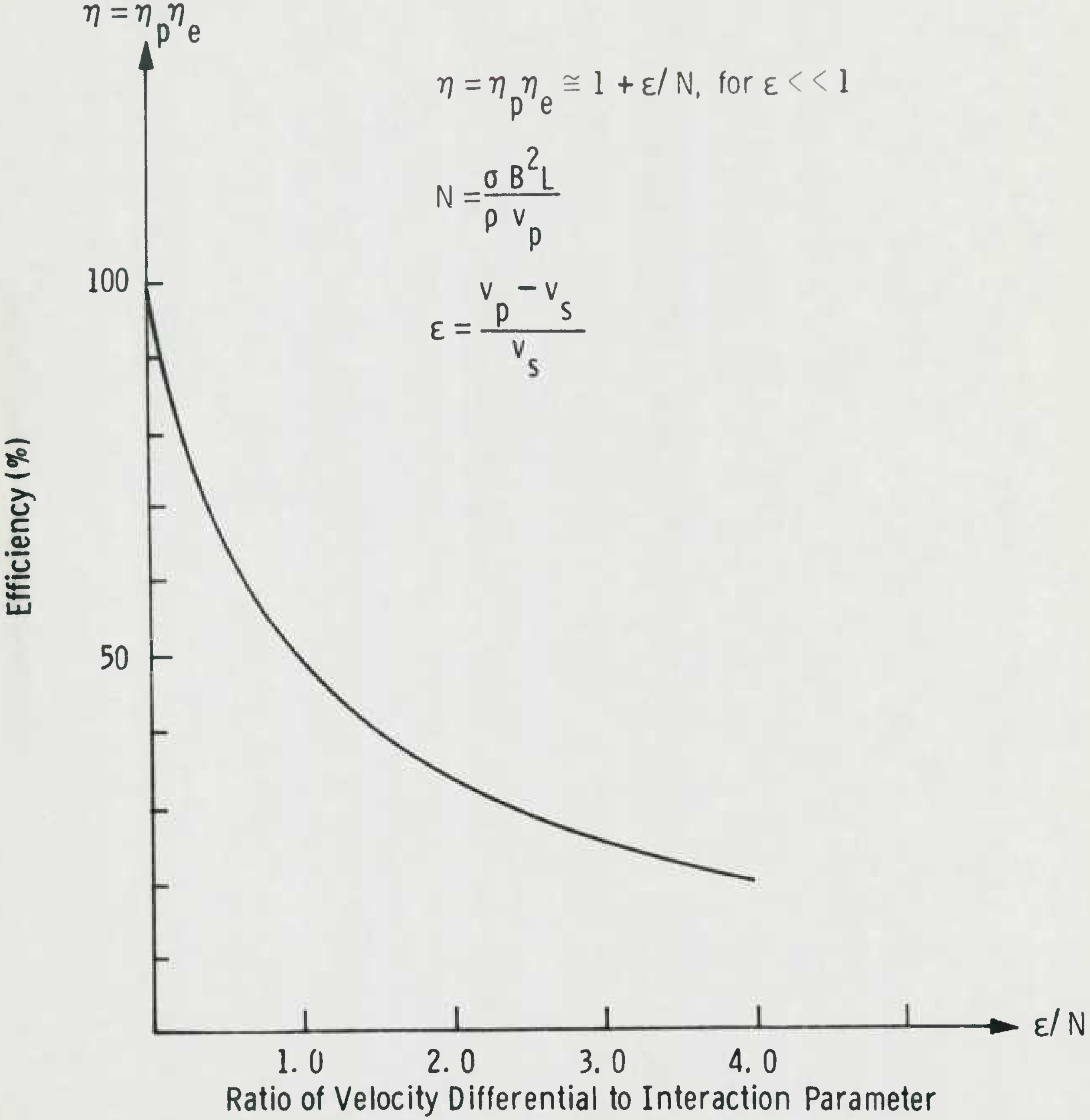


Fig. 32 — Sensitivity of ideal pump efficiency to velocity differential ÷ interaction parameter

Dwg. 7701A22

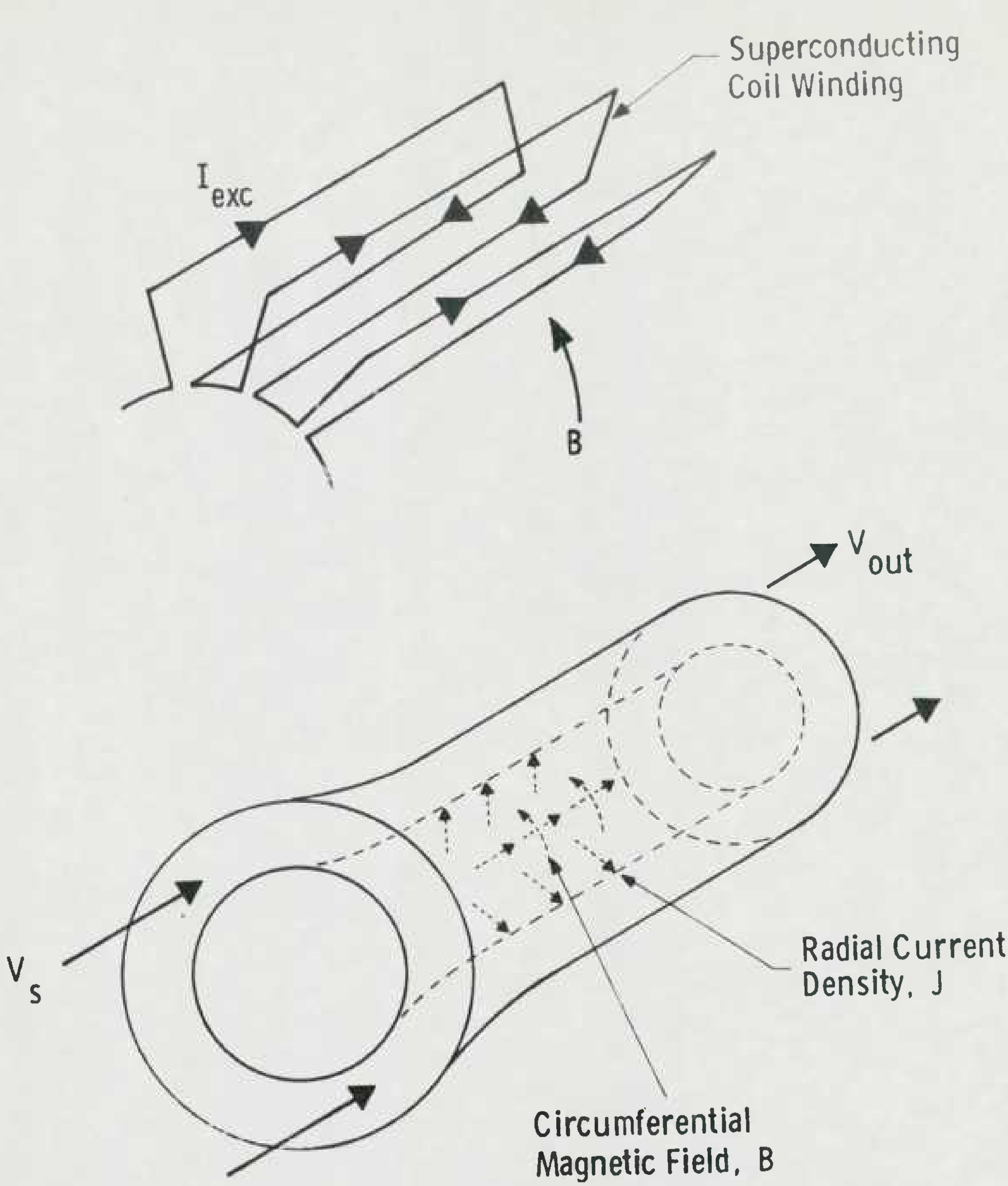


Fig. 33 – Annular pump configuration utilizing superconducting excitation. The toroidal winding establishes a high circumferential field in the pump region with low external flux leakage

APPENDIX A-1

SUBMERSIBLE HULL AND THRUSTER DRAG

The following calculations are presented to evaluate the frictional and total drag forces on a streamlined submersible of typical sizes and proportions moving at reasonable speeds, and to compare the frictional drag on a typical thruster to the total drag on the submersible. The frictional drags on the submersible and thruster are calculated by a standard velocity-squared formula that involves velocity-dependent parameters. However, the total drag characteristics used in Equation (5), Section 4.3, consist of a simple velocity-squared relationship and constant parameters. Therefore, discrepancies are likely to occur when comparing the total drag as given by Equation (5) to more accurate frictional drag calculations. The question is: "Are these discrepancies significant when compared to the total drag?"

Streamlined Hull Drag

The drag force, as used in the performance calculations in Section 4, is given by:

$$F_d = K_s v_s^2 \qquad (5)$$

where K_s was determined from supplied drag vs velocity data. In order to compare this to a more accurate frictional drag, we must specify the size, shape and, hence, total submerged surface area. To do this, we assume the hull shape may be approximated by a prolate spheroid whose total surface, S, is given by:

$$S = 2 \pi a^2 + (\pi b^2/\varepsilon) \cdot \ln [(1 + \varepsilon)/(1 - \varepsilon)] \qquad (A\text{-}1.1)$$

where 2a is the major axis, 2b is the minor axis, and ε is the eccentricity of the elliptical cross section:

$$\varepsilon = \sqrt{1 - (b/a)^2} \qquad \text{(A-1.2)}$$

For each hull diameter, we chose a length (and eccentricity) such that the calculated frictional drag at approximately 8 knots equals the drag given by Equation (5). The three hull sizes and shapes are summarized in the following table:

Hull Dia. (m)	Length (m)	Volume (m^3)	Surface Area (m^2)	Eccentricity	Displacement (Tons)
10.0	50.0	2620	4290	.980	2890
3.3	15.5	88.4	417	.977	97.4
1.0	4.35	2.28	33.2	.975	2.46

For each combination of submersible size and speed, the Reynolds' number for the resulting flow is calculated:

$$R = v_s \ell/\nu \qquad \text{(A-1.3)}$$

where v_s is the free-stream speed, ℓ is the length of the submersible, and ν is the kinematic viscosity of sea water. The coefficient of friction, C_F, is interpolated from a standard table[(7)] that lists C_F for various values of R. (It should be noted that all of the flows are turbulent; hence, the coefficient of friction decreases for increasing Reynolds' number.) Then, the frictional drag force is calculated:

$$F_f = 1/2 \, \rho \, C_F \, S \, v_s^2 \qquad \text{(A-1.4)}$$

where ρ is the density of sea water, C_F is the frictional coefficient, S is the wetted surface area (equal to the total surface area in all cases), and v_s is the free-stream speed of the submersible. Comparison between the simple velocity-squared drag used in Section 4 and the more accurate frictional drag is shown in Figure A1-1. As shown in the figure, values of these two forces agree to within twenty percent for each of the three sizes, over a speed range of up to 20 knots. We conclude that the approximations for C_D and S incorporated in the constant K_s are reasonable.

Thruster Frictional Drag

The calculations for frictional drag on a thruster duct are very similar to the hull drag calculations. For the purpose of making a specific hull to thruster drag comparison, the duct length was chosen as 2 m and the cross-sectional area as .25 m^2 (this does not include inlet shroud or outlet nozzle surfaces). For these calculations the aspect ratio, α, ranges from 1 to 20. The internal surface area is exactly:

$$S = 4 L a (1 + \alpha) \qquad \text{(A-1.5)}$$

where a is the duct half width and L is the length. The coefficient of friction is taken from the same data[7] for flow over a smooth flat plate (2-sides).

Frictional force is calculated for various speeds using Equation (A-1.4). Figure A-1.2 compares duct drag obtained using this formula to hull drag taken from Figure 1. As may be seen from the figure, an aspect ratio of 1 produces the minimum drag, and the drag per duct is less than ten percent of the total drag for the small hull and less than one percent of the total for the larger hull.

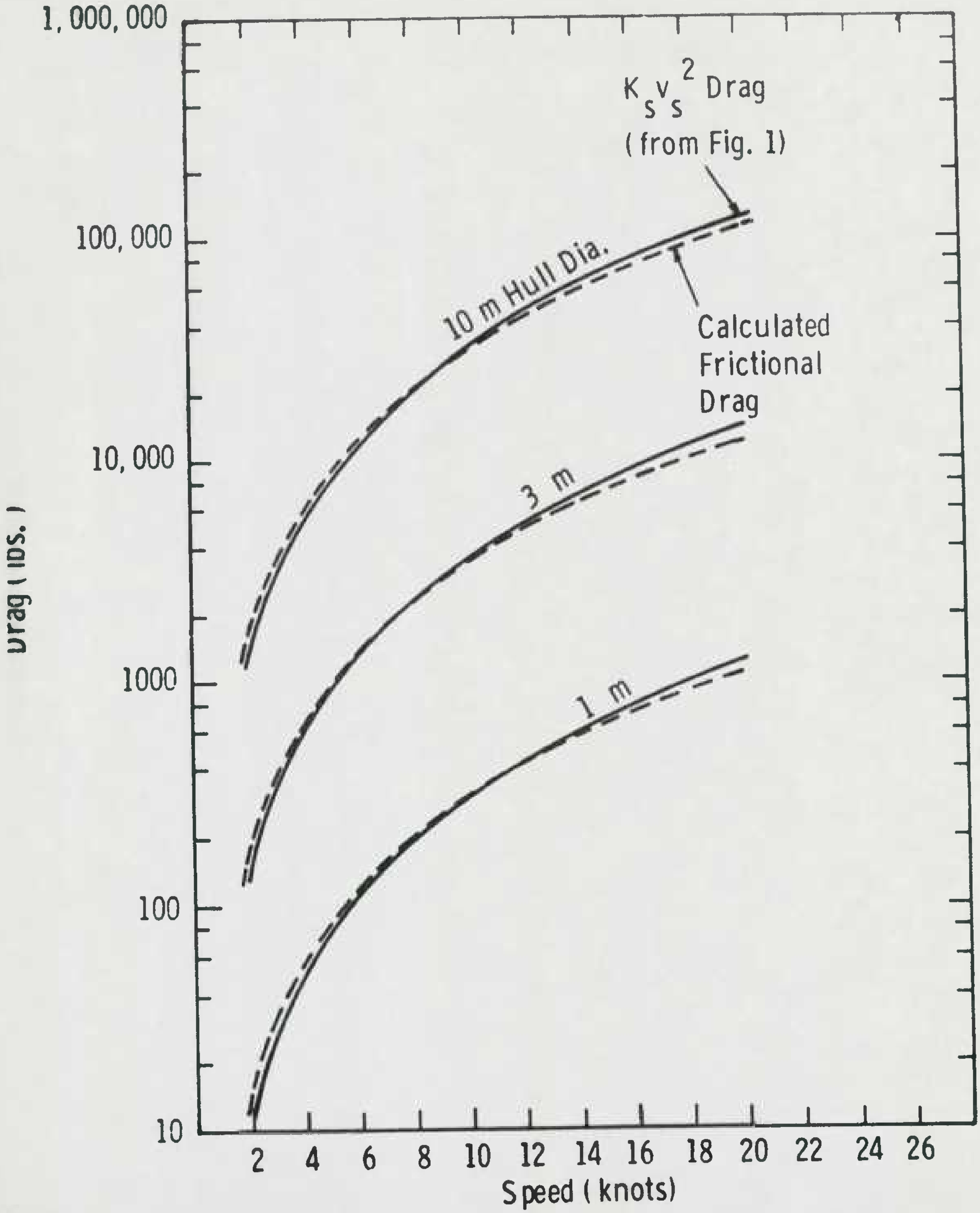


Fig. A-1.1 — Comparison of frictional hull drag to $K_s v_s^2$ drag

Curve 717912-A

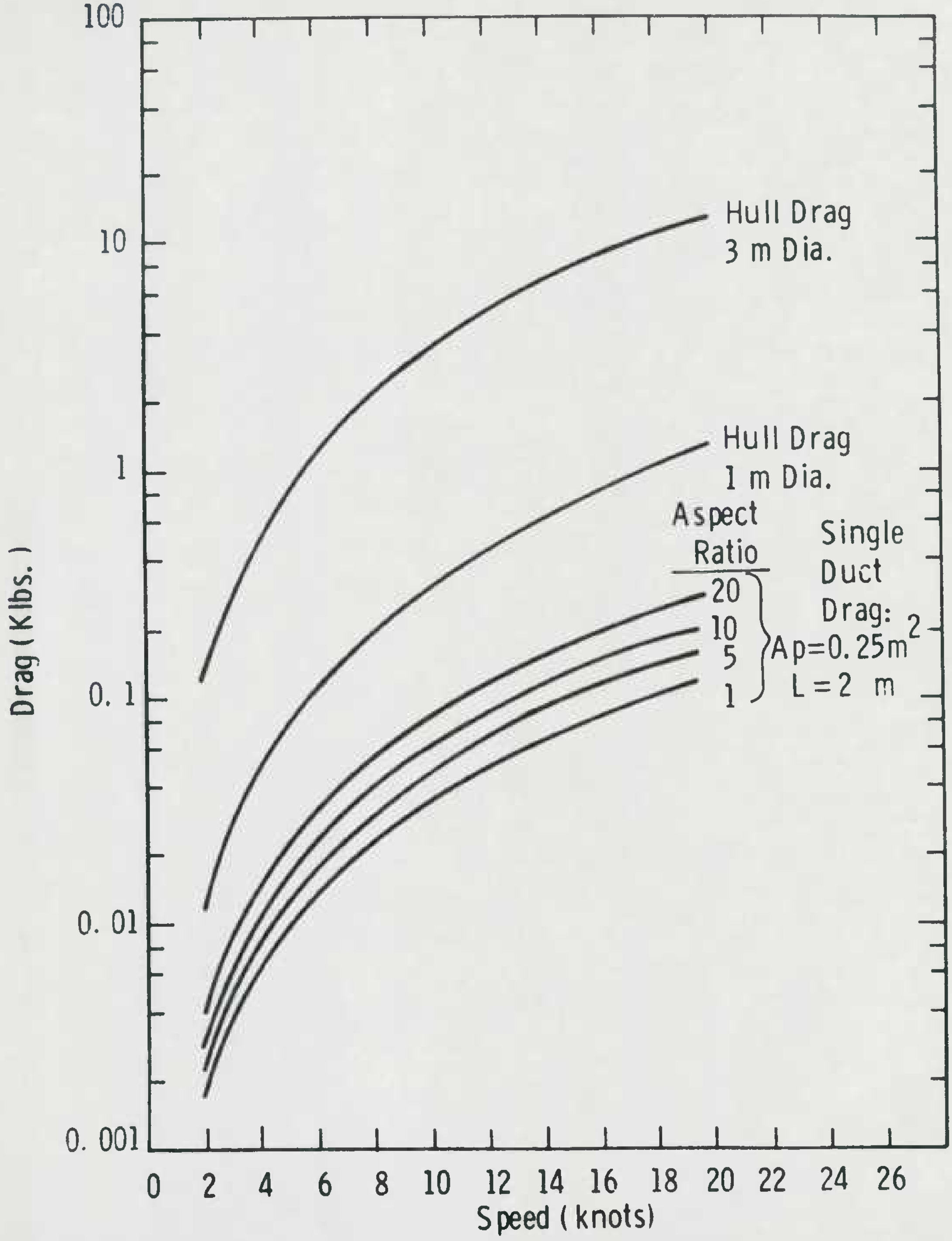


Fig. A-1. 2 — Comparison of thruster-duct drag to hull drag for selected duct size

APPENDIX A-2

ELECTROMAGNETIC COUPLING TO CONDUCTING FLUID FLOW

For fully developed incompressible flow, the vector equation of motion is given by:[8]

$$\bar{f}_b = \bar{\nabla} P - \mu_f \nabla^2 \bar{v} \qquad \text{(A-2.1)}$$

where $\bar{f}_b$ = body force

$\bar{\nabla} P$ = pressure gradient

$\mu_f \nabla^2 \bar{v}$ = viscous force density

Neglecting viscous forces and considering one-dimensional (x) motion, we have

$$\frac{\partial P}{\partial x} = (f_b)_x \qquad \text{(A-2.2)}$$

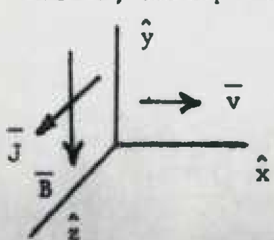
The electromagnetic body force density in a conducting fluid is given by

$$\bar{f}_b = \bar{J} \times \bar{B} \qquad \text{(A-2.3)}$$

where $\bar{J}$ = current density

$\bar{B}$ = magnetic field

If $\bar{J}$ and $\bar{B}$ are mutually perpendicular to the direction of fluid motion, as depicted in the sketch, where $\bar{J} = \hat{z} J$, $\bar{B} = -\hat{y} B$, then



$$\bar{f}_b = \hat{x} J B \qquad \text{(A-2.4)}$$

and from Equation (A-2.2), we have for the pressure gradient,

$$\frac{\partial P}{\partial x} = J\ B \qquad \text{(A-2.5)}$$

Assuming uniform current and magnetic field intensities, over electrode length L, we may integrate this expression to obtain for the pressure rise across the pump:

$$\Delta P_p = \int_o^L J\ B\ dx = J\ B\ L \qquad \text{(A-2.6)}$$

which is the desired expression relating pressure to current density, magnetic field, and electrode length.

Now we wish to relate current density to the difference in potential, V between the electrodes. Here we invoke Ohm's law in terms of a vector field equation for a moving medium.

$$\bar{J} = \sigma\ (\bar{E} + \bar{v} \times \bar{B}) \qquad \text{(A-2.7)}$$

where σ = conductivity of moving medium

$\bar{B}$ = magnetic field

$\bar{v}, \bar{E}$ = velocity and electric field as seen by stationary observer

Once again neglecting fringe effects, and separating the electrodes by a distance 2b along the z axis we have:

$$\bar{E} = \hat{z}\ V/2b\ , \quad \bar{v} \times \bar{B} = -\hat{z}\ v_p\ B$$

where v_p = $\hat{x}$ magnitude of velocity $\bar{v}$ so that

$$J = \sigma\ (V/2b - v_p\ B) \qquad \text{(A-2.8)}$$

Finally, substituting from Equation (A-2.6), we obtain Equation (14) used in Section 4.3.3:

$$\Delta P_p = \sigma\ B\ L\ (V/2b - v_p\ B) \qquad \text{(14)}$$

APPENDIX A-3

A3.1 Fortran Source Listing:

Calculations of ideal efficiency, voltage and current as function of:

- magnetic field
- area
- number of ducts
- hull size
- electrode length

A3.2 Sample Printout

A3.1 Source

```
@RUN, /RNPT SUBCAL,09B23NSFMD01,EMSUB,1,25

@DATA,IL      TPFS.
DATA 9R1 SL74T9 08/06/79 11:29:54 (->0)
                   DATA LINE /6/
                   REAL KO,K1,KS
                  DIMENSION EFFARY(3,4,20),XV(21),YV(21)
        C             EFFARY, XV,YV ALL USED FOR CALCOMP OUTPUT
        C               $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
        C
        C             VS IN METERS/SEC
        C
        C             AO IN SQ METERS
        C
        C             KO IN  NEWTONS/(M/S)**2
        C              LBS THRUST/KNOT**2 = KS*ALPHA
        C
        C             SPEED IN KNOTS    =  VS*BETA
        C
        C             RHO = DENSITY,KG/M**3
        C
        C             SIGMA = CONDUCTIVITY,MHOS
        C
        C
        C
                 KREF = 5000.
                 AO =.25
                 SIGMA = 4.
                  RHO = 1.E3
                  VPRIME = 1.
        C            VPRIME = KNOTS
                  ALPHA=(((2.57)**2)*6973.) /((31.E3)*25.)
                  BETA = 5./2.57
                  VSO=VPRIME/BETA
        C         VSO IN M/S
        C
        C
        C         JMAX     .*........# OF DUCTS
        C           LMAX...*..          VELOCITY
        C               MMAX........... BFIELD
        C          KMAX.....*.DUCT AREA
        C        NMAX.... ELECRRODE LENGTH
        C
                   JMAX = 4
                   LMAX = 19
                  KMAX=4
                  MMAX=3
                  NMAX=2
                  DO 100 M= 1,MMAX
                     IF(M.EQ.1) B = .5
                     IF(M.EQ.2) B = 2.
                     IF(M.GE.3) B = 5.
        C                DIST IN METERS
        C            DO 200 N = 1,NMAX
                       IF(N.EQ.1) DIST =2.
                       IF(N.GE.2) DIST=10.
        C
        C
        C
        C
                        DO 300 K= 1,KMAX
                            RK =K
                            A = AO*RK
                            SIDE =SQRT(A)
                   DO 400 JSIZ = 1,3
                   IF(JSIZ.EQ.1) KO = KREF
                      IF(JSIZ.EQ.2) KO = KREF/9.
                      IF(JSIZ.EQ.3) KO = KREF/100.
                    DO   400  J=1,JMAX
                        RJ=J
                        KS= KO/(RJ*RJ)
                        ALPHAK =ALPHA*KS
                           K1 = (1.+SQRT(1.+4.*KS/(RHO*A)))/2.
                           F  = K1*K1 - 1.
```

```
          WRITE(LINE,101)  KS,ALPHAK,A,K1
   101    FORMAT(1H ,/,'KS = 'E9.3,' NT/(M/S)2  = ',E9.3,' LB/(KT)2  ***',
        1 'AREA = ',F9.3,' SQ METERS',t***','K1 = ',
        2F9.3,'= RATIO VIN/VS',/)
          WRITE(LINE,103)
  103     FORMAT(1H ,5X,'VS(M/S)   VS(KNTS)   B(TESLA)  L(METERS) CUR(KAMP)
        1 VOLTS    PRESS(PSI) TH(KNTS)  TH(KLBS)  EFFICIENCY(%)',/)
C
C
          DO 500 L = 1,LMAX
C           COMPUTE  EVFICIENCY FOR VS= 2, 20 KTS
            RL = L*1
                      VS= VSO*RL
                      BETAVS = BETA*VS
C
C                  CONVERT  VS TO KNOTS
C
C
C
C
                          V2RO =(2.*SIGMA*(B**2)*DIST)/RHO
                          EFF  =(2.*KS/(A*RHO))/(F*(VS*F/V2RO +K1))

C               HERE WE ARE ASSUMING  ASPECT RATIO = 1 ,THEN
C
C                 AREA = SIDE**2
C
C
C
                          CUR = SIDE*RHO*(VS**2)*((K1**2)-1.)/B
C               USE SIDE/2 IN CURRENT EXPRESSION
                          CUR=CUR/2.
                          TH   = KS*VS**2
                          THPOS =TH*(6973./3)*E3)
                          VOLT= TH*VS/(CUR*EFF)
                          DELP = RHO*VS*VS*(K1*K1-1.)/2.
C          CONVERT DELP FROM NT/SQM TO PSI
                          PSI = DELP*1.45E-4

C
               WRITE(LINE,501) VS,BETAVS,B,DIST,CUR,VOLT,
         1PSI,TH,THPOS,EFF
  501     FORMAT(1H ,3X,4(F9.3,1X),-3PF9.3,1X,0PF9.3,1X,F10.2,1X
         12(-3PF9.3,1X),2PF9.3)
  500                     CONTINUE
  400              CONTINUE
C
C
          WRITE(LINE,301)
  301     FORMAT(1H ,/)
  300               CONTINUE
  200          CONTINUE
  100      CONTINUE
           STOP
           END
END DATA. ERRORS: NONE. TIME: 0.692 SEC. IMAGE COUNT: 124

@FIN
```

σ = 4.0 mhos/m

A3.2 Sample Printout

DATE 072779 PAGE 12

ELECTROMAGNETIC THRUST PROPULSION

Ks = .500+04 NT/(M/S)2 = .297+03 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 2.791= RATIO VIN/VS

Full size 1 Duct

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	7.177	898.550	.52	5.284	1.189	.084
1.542	3.000	.500	2.000	16.148	2020.662	1.17	11.889	2.674	.056
2.056	4.000	.500	2.000	28.708	3591.331	2.08	21.136	4.754	.042
2.570	5.000	.500	2.000	44.856	5610.559	3.25	33.024	7.428	.034
3.084	6.000	.500	2.000	64.592	8078.344	4.68	47.555	10.697	.028
3.598	7.000	.500	2.000	87.917	10994.686	6.37	64.728	14.560	.024
4.112	8.000	.500	2.000	114.831	14359.587	8.33	84.543	19.017	.021
4.626	9.000	.500	2.000	145.333	18173.045	10.54	106.999	24.068	.019
5.140	10.000	.500	2.000	179.423	22435.061	13.01	132.098	29.714	.017
5.654	11.000	.500	2.000	217.102	27145.635	15.74	159.839	35.953	.015
6.168	12.000	.500	2.000	258.369	32304.766	18.73	190.221	42.787	.014
6.682	13.000	.500	2.000	303.225	37912.455	21.98	223.246	50.216	.013
7.196	14.000	.500	2.000	351.669	43968.703	25.50	258.912	58.239	.012
7.710	15.000	.500	2.000	403.702	50473.506	29.27	297.220	66.855	.011
8.224	16.000	.500	2.000	459.323	57426.870	33.30	338.171	76.067	.011
8.738	17.000	.500	2.000	518.533	64828.790	37.59	381.763	85.872	.010
9.252	18.000	.500	2.000	581.331	72679.267	42.15	427.997	96.272	.009
9.766	19.000	.500	2.000	647.717	80978.303	46.96	476.874	107.266	.009
10.280	20.000	.500	2.000	717.692	89725.896	52.03	528.392	118.854	.008

Ks = .125+04 NT/(M/S)2 = .743+02 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.725= RATIO VIN/VS

4 Ducts

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	2.087	261.746	.15	1.321	.297	.249
1.542	3.000	.500	2.000	4.695	588.264	.34	2.972	.669	.166
2.056	4.000	.500	2.000	8.348	1045.212	.61	5.284	1.189	.125
2.570	5.000	.500	2.000	13.043	1632.590	.95	8.256	1.857	.100
3.084	6.000	.500	2.000	18.782	2350.398	1.36	11.889	2.674	.083
3.598	7.000	.500	2.000	25.564	3198.636	1.85	16.182	3.640	.071
4.112	8.000	.500	2.000	33.390	4177.303	2.42	21.136	4.754	.062
4.626	9.000	.500	2.000	42.259	5286.401	3.06	26.750	6.017	.055
5.140	10.000	.500	2.000	52.172	6525.928	3.78	33.024	7.428	.050
5.654	11.000	.500	2.000	63.128	7895.880	4.58	39.960	8.988	.045
6.168	12.000	.500	2.000	75.128	9396.273	5.45	47.555	10.697	.042
6.682	13.000	.500	2.000	88.171	11027.090	6.39	55.811	12.554	.038
7.196	14.000	.500	2.000	102.257	12788.[illegible]	7.41	64.728	14.560	.036
7.710	15.000	.500	2.000	117.387	14680.015	8.51	74.305	16.714	.033
8.224	16.000	.500	2.000	133.560	16702.123	9.68	84.543	19.017	.031
8.738	17.000	.500	2.000	150.777	18854.659	10.93	95.441	21.468	.029
9.252	18.000	.500	2.000	169.037	21137.625	12.26	106.999	24.068	.028
9.766	19.000	.500	2.000	188.341	23551.022	13.65	119.218	26.816	.026
10.280	20.000	.500	2.000	208.688	26094.248	15.13	132.098	29.714	.025

Ks = .556+03 NT/(M/S)2 = .330+02 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.398= RATIO VIN/VS

9 Ducts

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	1.007	126.619	.07	.587	.132	.473
1.542	3.000	.500	2.000	2.266	284.353	.16	1.321	.297	.316
2.056	4.000	.500	2.000	4.029	505.035	.29	2.348	.528	.237
2.570	5.000	.500	2.000	6.295	788.673	.46	3.669	.825	.190
3.084	6.000	.500	2.000	9.065	1135.258	.66	5.284	1.189	.158
3.598	7.000	.500	2.000	12.338	1544.793	.89	7.192	1.618	.136
4.112	8.000	.500	2.000	16.115	2017.279	1.17	9.394	2.113	.119
4.626	9.000	.500	2.000	20.396	2552.715	1.48	11.889	2.674	.106
5.140	10.000	.500	2.000	25.180	3151.100	1.83	14.678	3.302	.095
5.654	11.000	.500	2.000	30.468	3812.436	2.21	17.760	3.995	.086
6.168	12.000	.500	2.000	36.259	4536.723	2.63	21.136	4.754	.079
6.682	13.000	.500	2.000	42.554	5323.959	3.09	24.805	5.580	.073
7.196	14.000	.500	2.000	49.353	6174.146	3.58	28.768	6.471	.068
7.710	15.000	.500	2.000	56.655	7087.282	4.11	33.024	7.428	.063
8.224	16.000	.500	2.000	64.461	8063.369	4.67	37.575	8.452	.059
8.738	17.000	.500	2.000	72.770	9102.406	5.28	42.418	9.541	.056
9.252	18.000	.500	2.000	81.583	10204.393	5.91	47.555	10.697	.053
9.766	19.000	.500	2.000	90.900	11369.331	6.59	52.986	11.918	.050
10.280	20.000	.500	2.000	100.720	12597.218	7.30	58.710	13.206	.048

Ks = .312+03 NT/(M/S)2 = .186+02 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.250= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)

1.028	2.000	.500	[illegible]	[illegible]	74.948	.04	.330	.074	.762
1.542	3.000	.500	[illegible]	[illegible]	168.150	.10	[illegible]	.167	.509
2.056	4.000	.500	[illegible]	[illegible]	298.505	.17	1.321	.297	.383
2.570	5.000	.500	[illegible]	[illegible]	466.013	[illegible]	[illegible]	.464	.306
3.084	6.000	.500	[illegible]	[illegible]	670.674	[illegible]	[illegible]	.669	.255
3.598	7.000	.500	[illegible]	[illegible]	912.486	[illegible]	[illegible]	.910	.219
4.112	8.000	.500	[illegible]	[illegible]	1191.452	[illegible]	[illegible]	1.189	.192
4.626	9.000	.500	[illegible]	12.037	1507.570	[illegible]	[illegible]	1.504	.170
5.140	10.000	.500	[illegible]	[illegible]	1860.841	[illegible]	[illegible]	1.857	.153
5.654	11.000	.500	[illegible]	[illegible]	2251.264	[illegible]	[illegible]	2.247	.140
6.168	12.000	.500	[illegible]	[illegible]	2678.839	[illegible]	11.889	2.674	.128
6.682	13.000	.500	[illegible]	[illegible]	3143.568	[illegible]	13.953	3.138	.118
7.196	14.000	.500	[illegible]	[illegible]	3645.448	[illegible]	[illegible]	3.640	.110
7.710	15.000	.500	[illegible]	[illegible]	4184.482	[illegible]	18.576	4.178	.102
8.224	16.000	.500	[illegible]	[illegible]	4760.668	[illegible]	21.136	4.754	.096
8.738	17.000	.500	[illegible]	[illegible]	5374.006	[illegible]	[illegible]	5.367	.090
9.252	18.000	.500	[illegible]	[illegible]	6024.497	[illegible]	[illegible]	6.017	.085
9.766	19.000	.500	[illegible]	[illegible]	6712.141	[illegible]	[illegible]	6.704	.081
10.280	20.000	.500	2.000	[illegible]	7436.937	[illegible]	33.024	7.428	.077

16 DUCTS

Ks = .556+03 NT/(M/S)2 = .330+02 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.398= RATIO VIN/VS

THIRD SIZE 1 DUCT

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	1.007	126.619	.07	.587	.132	.473
1.542	3.000	.500	2.000	2.266	284.353	.16	1.321	.297	.316
2.056	4.000	.500	2.000	4.029	505.038	.27	2.348	.528	.237
2.570	5.000	.500	2.000	6.295	788.673	.46	3.669	.825	.190
3.084	6.000	.500	2.000	9.065	1135.258	.66	5.284	1.189	.158
3.598	7.000	.500	2.000	12.338	1544.793	.89	7.192	1.618	.136
4.112	8.000	.500	2.000	16.115	2017.279	1.17	9.394	2.113	.119
4.626	9.000	.500	2.000	20.396	2552.715	1.48	11.889	2.674	.106
5.140	10.000	.500	2.000	25.180	3151.100	1.83	14.678	3.302	.095
5.654	11.000	.500	2.000	30.468	3812.436	2.21	17.760	3.995	.086
6.168	12.000	.500	2.000	[illegible]	4536.723	2.63	21.136	4.754	.079
6.682	13.000	.500	2.000	[illegible]	5323.959	3.09	24.805	5.580	.073
7.196	14.000	.500	2.000	[illegible]	6174.146	3.58	28.768	6.471	.068
7.710	15.000	.500	2.000	[illegible]	7087.282	4.11	33.024	7.428	.063
8.224	16.000	.500	2.000	[illegible]	8063.369	4.67	37.575	8.452	.059
8.738	17.000	.500	2.000	[illegible]	9102.406	5.28	42.418	9.541	.056
9.252	18.000	.500	2.000	81.583	10204.393	5.91	47.555	10.697	.053
9.766	19.000	.500	2.000	90.900	11369.331	6.59	52.986	11.918	.050
10.280	20.000	.500	2.000	100.720	12597.218	7.30	58.710	13.206	.048

Ks = .139+03 NT/(M/S)2 = .825+01 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.124= RATIO VIN/VS

4 DUCTS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.277	35.253	.02	.147	.033	1.543
1.542	3.000	.500	2.000	.624	78.886	.05	.330	.074	1.034
2.056	4.000	.500	2.000	1.110	139.857	.08	.587	.132	.778
2.570	5.000	.500	2.000	1.734	218.166	.13	.917	.206	.623
3.084	6.000	.500	2.000	2.497	313.812	.18	1.321	.297	.520
3.598	7.000	.500	2.000	3.398	426.796	.25	1.798	.404	.446
4.112	8.000	.500	2.000	4.438	557.118	.32	2.348	.528	.391
4.626	9.000	.500	2.000	5.617	704.778	.41	2.972	.669	.347
5.140	10.000	.500	2.000	6.935	869.776	.50	3.669	.825	.313
5.654	11.000	.500	2.000	8.391	1052.111	.61	4.440	.999	.284
6.168	12.000	.500	2.000	9.987	1251.784	.72	5.284	1.189	.261
6.682	13.000	.500	2.000	11.720	1468.795	.85	6.201	1.395	.241
7.196	14.000	.500	2.000	13.593	1703.143	.99	7.192	1.618	.224
7.710	15.000	.500	2.000	15.604	1954.829	1.13	8.256	1.857	.209
8.224	16.000	.500	2.000	17.754	2223.853	1.29	9.394	2.113	.196
8.738	17.000	.500	2.000	20.042	2510.215	1.45	10.605	2.385	.184
9.252	18.000	.500	2.000	22.470	2813.915	1.63	11.889	2.674	.174
9.766	19.000	.500	2.000	25.036	3134.952	1.82	13.246	2.980	.165
10.280	20.000	.500	2.000	27.740	3473.327	2.01	14.675	3.302	.157

Ks = .617+02 NT/(M/S)2 = .367+01 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.058= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.127	16.403	.01	.065	.015	3.222
1.542	3.000	.500	2.000	.285	36.499	.02	.147	.033	2.172
2.056	4.000	.500	2.000	.507	64.524	.04	.261	.059	1.638
2.570	5.000	.500	2.000	.793	100.479	.06	.408	.092	1.315

9 DUCTS

3.084	6.000	.500	2.000	1.142	144.363	.08	.587	.132	1.098
3.598	7.000	.500	2.000	1.554	196.177	.11	.799	.180	.943
4.112	8.000	.500	2.000	2.030	255.920	.15	1.044	.235	.826
4.626	9.000	.500	2.000	2.569	323.593	.19	1.321	.297	.735
5.140	10.000	.500	2.000	3.172	399.195	.23	1.631	.367	.662
5.654	11.000	.500	2.000	3.838	482.727	.28	1.973	.444	.602
6.168	12.000	.500	2.000	4.567	574.188	.33	2.348	.528	.552
6.682	13.000	.500	2.000	5.360	673.579	.39	2.756	.620	.510
7.196	14.000	.500	2.000	6.217	780.899	.45	3.196	.719	.474
7.710	15.000	.500	2.000	7.137	896.169	.52	3.669	.825	.442
8.224	16.000	.500	2.000	8.120	1019.328	.59	4.175	.939	.415
8.738	17.000	.500	2.000	9.167	1150.437	.66	4.713	1.060	.391
9.252	18.000	.500	2.000	10.277	1289.475	.75	5.284	1.189	.369
9.766	19.000	.500	2.000	11.450	1436.443	.83	5.887	1.324	.350
10.280	20.000	.500	2.000	12.687	1591.340	.92	6.523	1.467	.332

K_S = .347+02 NT/(M/S)2 = .206+01 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.034= RATIO VIN/VS

16 DUCTS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.072	9.556	.01	.037	.008	5.468
1.542	3.000	.500	2.000	.162	21.102	.01	.083	.019	3.714
2.056	4.000	.500	2.000	.289	37.160	.02	.147	.033	2.812
2.570	5.000	.500	2.000	.451	57.731	.03	.229	.052	2.263
3.084	6.000	.500	2.000	.650	82.813	.05	.330	.074	1.893
3.598	7.000	.500	2.000	.884	112.408	.06	.450	.101	1.627
4.112	8.000	.500	2.000	1.155	146.515	.08	.587	.132	1.426
4.626	9.000	.500	2.000	1.462	185.135	.11	.743	.167	1.270
5.140	10.000	.500	2.000	1.805	228.266	.13	.917	.206	1.144
5.654	11.000	.500	2.000	2.184	275.910	.16	1.110	.250	1.042
6.168	12.000	.500	2.000	2.599	328.066	.19	1.321	.297	.956
6.682	13.000	.500	2.000	3.050	384.734	.22	1.550	.349	.883
7.196	14.000	.500	2.000	3.538	445.914	.26	1.798	.404	.820
7.710	15.000	.500	2.000	4.061	511.607	.29	2.064	.464	.766
8.224	16.000	.500	2.000	4.620	581.811	.33	2.348	.528	.718
8.738	17.000	.500	2.000	5.216	656.528	.38	2.651	.596	.676
9.252	18.000	.500	2.000	5.848	735.757	.42	2.972	.669	.639
9.766	19.000	.500	2.000	6.516	819.498	.47	3.312	.745	.606
10.280	20.000	.500	2.000	7.220	907.752	.52	3.669	.825	.576

K_S = .500+02 NT/(M/S)2 = .297+01 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.048= RATIO VIN/VS

TENTH-SIZE 1 DUCT

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.103	13.447	.01	.053	.012	3.911
1.542	3.000	.500	2.000	.232	29.853	.02	.119	.027	2.643
2.056	4.000	.500	2.000	.413	52.715	.03	.211	.048	1.996
2.570	5.000	.500	2.000	.645	82.027	.05	.330	.074	1.603
3.084	6.000	.500	2.000	.929	117.796	.07	.476	.107	1.340
3.598	7.000	.500	2.000	1.265	160.019	.09	.647	.146	1.150
4.112	8.000	.500	2.000	1.652	208.697	.12	.845	.190	1.008
4.626	9.000	.500	2.000	2.091	263.830	.15	1.070	.241	.897
5.140	10.000	.500	2.000	2.582	325.416	.19	1.321	.297	.808
5.654	11.000	.500	2.000	3.124	393.458	.23	1.598	.360	.735
6.168	12.000	.500	2.000	3.718	467.953	.27	1.902	.428	.674
6.682	13.000	.500	2.000	4.363	548.904	.32	2.232	.502	.623
7.196	14.000	.500	2.000	5.060	636.308	.37	2.589	.582	.579
7.710	15.000	.500	2.000	5.809	730.167	.42	2.972	.669	.540
8.224	16.000	.500	2.000	6.609	830.481	.48	3.382	.761	.507
8.738	17.000	.500	2.000	7.461	937.249	.54	3.818	.859	.477
9.252	18.000	.500	2.000	8.365	1050.472	.61	4.280	.963	.451
9.766	19.000	.500	2.000	9.320	1170.149	.68	4.769	1.073	.427
10.280	20.000	.500	2.000	10.327	1296.280	.75	5.284	1.189	.406

K_S = .125+02 NT/(M/S)2 = .743+00 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.012= RATIO VIN/VS

4 DUCTS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.026	3.803	.00	.013	.003	13.600
1.542	3.000	.500	2.000	.059	8.166	.00	.030	.007	9.500
2.056	4.000	.500	2.000	.105	14.170	.01	.053	.012	7.299
2.570	5.000	.500	2.000	.164	21.815	.01	.083	.019	5.927
3.084	6.000	.500	2.000	.236	31.102	.02	.119	.027	4.988
3.598	7.000	.500	2.000	.322	42.029	.02	.162	.036	4.307
4.112	8.000	.500	2.000	.420	54.598	.03	.211	.048	3.789
4.626	9.000	.500	2.000	.532	68.808	.04	.267	.060	3.382

5.140	10.000	.500	2.000	.656	84.639	.05	.330	.074	3.054
5.654	11.000	.500	2.000	.781	102.152	.06	.400	.090	2.784
6.168	12.000	.500	2.000	.943	121.285	.07	.476	.107	2.558
6.682	13.000	.500	2.000	1.104	142.060	.08	.558	.126	2.366
7.196	14.000	.500	2.000	1.287	164.476	.09	.647	.146	2.201
7.710	15.000	.500	2.000	1.477	188.532	.11	.743	.167	2.057
8.224	16.000	.500	2.000	1.681	214.590	.12	.843	.190	1.931
8.738	17.000	.500	2.000	1.897	241.570	.14	.954	.215	1.820
9.252	18.000	.500	2.000	2.127	270.550	.15	1.070	.241	1.720
9.766	19.000	.500	2.000	2.370	301.172	.17	1.192	.268	1.631
10.280	20.000	.500	2.000	2.626	333.434	.19	1.321	.297	1.551

Ks = .556+01 NT/(M/S)2 = .330+00 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.006= RATIO VIN/VS

9 DUCTS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.012	1.981	[illegible]	.006	.001	26.024
1.542	3.000	.500	2.000	.026	4.069	[illegible]	.013	.003	19.002
2.056	4.000	.500	2.000	.047	6.889	.00	.023	.005	14.964
2.570	5.000	.500	2.000	.073	10.440	.01	.037	.008	12.342
3.084	6.000	.500	2.000	.105	14.724	.01	.053	.012	10.502
3.598	7.000	.500	2.000	.143	19.740	.01	.072	.016	9.139
4.112	8.000	.500	2.000	.187	25.487	.01	.094	.021	[illegible]
4.626	9.000	.500	2.000	.237	31.968	.02	.119	.027	[illegible]
5.140	10.000	.500	2.000	.293	39.179	.02	.147	.033	[illegible]
5.654	11.000	.500	2.000	.354	47.120	.03	.178	.040	[illegible]
6.168	12.000	.500	2.000	.422	55.795	.03	.211	.048	[illegible]
6.682	13.000	.500	2.000	.495	65.206	.04	.248	.056	5.138
7.196	14.000	.500	2.000	.574	75.560	.04	.288	.065	4.789
7.710	15.000	.500	2.000	.659	86.211	.05	.330	.074	4.484
8.224	16.000	.500	2.000	.749	97.813	.05	.376	.085	4.216
8.738	17.000	.500	2.000	.846	110.147	.06	.424	.095	3.977
9.252	18.000	.500	2.000	.948	123.213	.07	.476	.107	3.765
9.766	19.000	.500	2.000	1.057	137.011	.08	.530	.119	3.574
10.280	20.000	.500	2.000	1.171	151.540	.08	.587	.132	3.401

≈ Ks = .312+01 NT/(M/S)2 = .186+00 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.003= RATIO VIN/VS

16 DUCTS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.007	1.360	.00	.003	.001	38.420
1.542	3.000	.500	2.000	.015	2.628	.00	.007	.002	29.382
2.056	4.000	.500	2.000	.026	4.329	.00	.013	.003	23.787
2.570	5.000	.500	2.000	.041	6.441	.00	.021	.005	19.981
3.084	6.000	.500	2.000	.058	8.966	.00	.030	.007	17.226
3.598	7.000	.500	2.000	.081	11.903	.01	.040	.009	15.138
4.112	8.000	.500	2.000	.106	15.252	.01	.053	.012	13.502
4.626	9.000	.500	2.000	.134	19.015	.01	.067	.015	12.184
5.140	10.000	.500	2.000	.165	23.186	.01	.083	.019	11.101
5.654	11.000	.500	2.000	.199	27.775	.01	.100	.022	10.195
6.168	12.000	.500	2.000	.237	32.769	.02	.119	.027	9.426
6.682	13.000	.500	2.000	.279	38.179	.02	.140	.031	8.764
7.196	14.000	.500	2.000	.323	44.001	.02	.162	.036	8.190
7.710	15.000	.500	2.000	.371	50.235	.03	.186	.042	7.686
8.224	16.000	.500	2.000	.422	56.882	.03	.211	.048	7.240
8.738	17.000	.500	2.000	.476	63.940	.03	.239	.054	6.844
9.252	18.000	.500	2.000	.534	71.411	.04	.267	.060	6.488
9.766	19.000	.500	2.000	.595	79.294	.04	.298	.067	6.168
10.280	20.000	.500	2.000	.659	87.589	.05	.330	.074	5.877

Ks = .500+04 NT/(M/S)2 = .297+03 LB/(KT)2 ***AREA = .250 SQ METERS***K1 = 5.000= RATIO VIN/VS

PLOT #5

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	10.000	12.681	318.320	1.84	5.284	1.189	.135
1.542	3.000	.500	10.000	28.533	715.257	4.14	11.889	2.674	.090
2.056	4.000	.500	10.000	50.726	1270.711	7.36	21.136	4.754	.067
2.570	5.000	.500	10.000	79.259	1984.682	11.49	33.024	7.428	.054
3.084	6.000	.500	10.000	114.133	2857.172	16.55	47.555	10.697	.045
3.598	7.000	.500	10.000	155.347	3888.179	22.53	64.728	14.560	.039
4.112	8.000	.500	10.000	202.903	5077.703	29.42	84.543	19.017	.034
4.626	9.000	.500	10.000	256.798	6425.745	37.24	106.999	24.068	.030
5.140	10.000	.500	10.000	317.035	7932.305	45.97	132.098	29.714	.027
5.654	11.000	.500	10.000	383.613	9597.382	55.62	159.839	35.953	.025

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